

PROGRAMME

8th International Meeting on Origami in Science, Mathematics and Education (80SME)

16 – 18 July 2024 Melbourne, Australia

Published by Organising Committee 8th International Meeting on Origami in Science, Mathematics and Education (8OSME) 16–18 July 2024 Melbourne, Australia

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Welcome Message

It is our great pleasure and honour to host the 8th International Meeting on Origami in Science, Mathematics and Education (8OSME) at Swinburne University of Technology in Melbourne, Australia, 16-18 July 2024. 8OSME will be held in conjunction with the Folding Australia 2024 origami convention on 20-21 July.

The OSME meeting is one of the most important gatherings of the global origami community. Previous meetings, held approximately every four years, have been highly successful. The meeting proceedings have become the definitive guide for advanced research in origami and its applications.

There were 191 abstracts submitted for this conference, including 127 full papers. We have six exemplary keynote speakers for plenary sessions and four parallel sessions over the three days, with an extremely varied programme focused on the themes of Engineering, Math, Computation, History, Education and Design.

80SME offers opportunities for researchers and practitioners to exchange the latest development in this important area. We wish you a fruitful conference. Furthermore, Melbourne offers a plethora of options for leisure, entertainment, and a rich cultural tapestry. We trust you will have a memorable experience and a happy stay in our city.

80SME Organising Committee

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80SME Organising Committee

Chairs:

Guoxing Lu (Chair)	Swinburne University of Technology (Formerly)
Michael Assis (Vice-Chair)	University of Melbourne
	Melbourne Origami Group
Jianjun Zhang (Vice-Chair)	Swinburne University of Technology

Organising Committee Members:

Winnie Leung	Sydney Origami Inc.
Malte Wagenfeld	RMIT University
Sachiko Ishida	Meiji University
Sukanya Deshmukh	RMIT University
Dian Zhang	Swinburne University of Technology (Formerly)
Xi Zhang	Swinburne University of Technology (Formerly)
Chengzheng Mao	Swinburne University of Technology

Steering Committee:

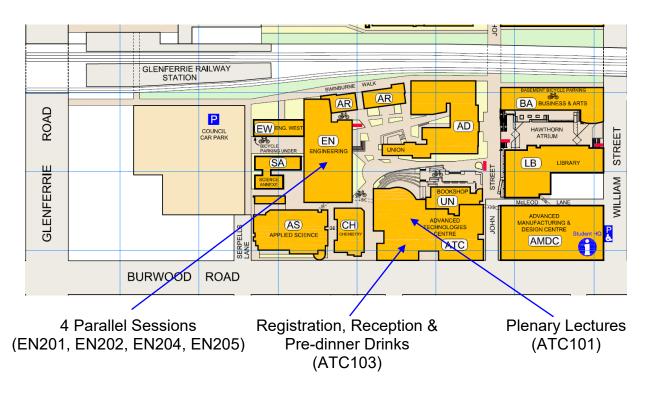
Emma Frigerio (1OST)	Emeritus Professor
Toshikazu Kawasaki (2OSSO)	Emeritus Professor
Thomas Hull (3OSME)	Franklin & Marshall College
Robert J. Lang (4OSME)	Independent artist and consultant
Patsy Wang-Iverson (5OSME)	Independent consultant
Tomohiro Tachi (6OSME)	University of Tokyo
Zhong You (7OSME)	University of Oxford

Scientific Committee

Jianguo Cai	Southeast University
Yan Chen	Tianjin University
Joseph Choma	Florida Atlantic University
Matthew Gardiner	Ars Electronica Futurelab
Joe Gattas	University of Queensland
Miri Golan	Israeli Origami Centre
Simon Guest	University of Cambridge
Hanqing Jiang	Westlake University
Paul Jackson	Shenkar College
Yves Klett	Foldcore GmbH
Jun Mitani	University of Tsukuba
Glaucio Paulino	Princeton University
Mark Schenk	University of Bristol
Tomohiro Tachi	University of Tokyo
Patsy Wang-Iverson	Independent Consultant
Zhong You (Chair)	University of Oxford

General Information

Conference venue floor plan



Hawthorn Campus, Swinburne University of Technology (425 Burwood Rd, Hawthorn VIC 3122)

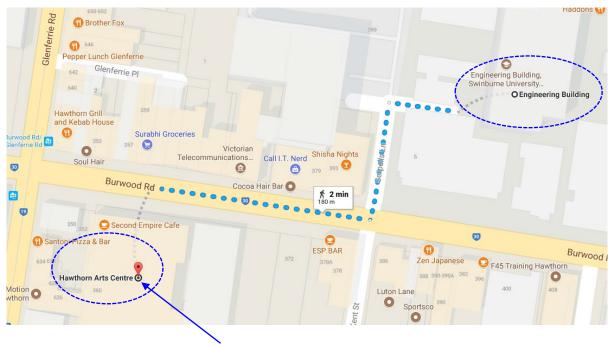
Conference Banquet

Date: 17 July 2024

Time: Pre-dinner drinks (ATC103 at Swinburne): 18:30 – 19:30

Conference Banquet (Chandelier Room): 19:30 - 23:00

Banquet Venue: Chandelier Room (Ground Floor) at Hawthorn Arts Centre



Conference Banquet (Hawthorn Arts Centre)

80SME Conference Organising Committee

Email: 80sme@swin.edu.au

Overview for 80SME

Date	Event	Time	Details	Chair	Venue
15 Jul	Registration	16:00-18:00	_	—	ATC103
10 001	Welcome Reception	18:00-19:30	Cocktail Reception	Prof Guoxing Lu	ATOTOS
	Registration	8:00-9:00	_	—	ATC103
	Opening Ceremony	9:00-9:10	Prof Emad Gad (Dean, School of Engineering, Swinburne University of Technology)	Prof Guoxing Lu	
	Plenary Lecture 1	9:10-10:00	Darryl Bedford (Drawstring Origami LIK)		ATC101
	Plenary Lecture 2	10:00-10:50	Prof Yan Chen (Tianjin University, China) Modular Origami: Kinematics and Metamaterials	Ŭ	
	Tea/Coffee Break	10:50-11:30	_	—	ATC103
			See programme for Engineering 1	—	EN201
	Parallel Sessions	11:30-12:30	See programme for Engineering 2	—	EN202
10 Jul		0000012.00	See programme for Mathematics, Computation & History	—	EN204
(Day 1)			See programme for Design & Education	—	EN205
	Lunch	12:30-14:00	_	—	ATC103
			See programme for Engineering 1	—	EN201
	Parallel Sessions	14:00-16:00	See programme for Engineering 2	—	EN202
	Parallel Sessions		See programme for Mathematics, Computation & History	—	EN204
	See programme for Design & Education		—	EN205	
	Tea/Coffee Break	16:00-16:30	_	—	ATC103
		See programme for Engineering 1	—	EN201	
	Parallel Sessions	16:30-18:10	See programme for Engineering 2	—	EN202
		10.30-16.10	See programme for Mathematics & Computation & History	—	EN204
			See programme for Design & Education	—	EN205

Date	Event	Time	Details	Chair	Venue	
	Registration	8:00-9:00	—	—	ATC103	
	Plenary Lecture 3	9:00-9:50	Distinguished Prof David Eppstein (University of California, Irvine, Canada) Computational Complexity and Parameterized Complexity of Folding	Prof Yimin Xie	ATC101	
	Plenary Lecture 4	9:50-10:40	Tomoko Fuse (Origami Artist, Japan) Infinite Fold and Repeated Fold			
	Group Photo	10:40-11:00	— — — — — — — — — — — — — — — — — — —	—	ATC Foyer	
	Tea/Coffee Break	11:00-11:30	Tea/Coffee Break	—	ATC103	
			See programme for Engineering 1	—	EN201	
	Parallel Sessions	11:30-12:30	See programme for Engineering 2	—	EN202	
	Falallel Sessions	11.30-12.30	See programme for Mathematics, Computation & History	—	EN204	
17 Jul			See programme for Design & Education	—	EN205	
(Day 2)	Lunch	12:30-14:00	_	—	ATC103	
			See programme for Engineering 1	—	EN201	
	Parallel Sessions	14:00-16:00	See programme for Engineering 2	—	EN202	
	Falallel Sessions	14.00-10.00	See programme for Mathematics, Computation & History	—	EN204	
			See programme for Design & Education	_	EN205	
	Tea/Coffee Break	16:00-16:30	—	—	ATC103	
			See programme for Engineering 1	—	EN201	
	Parallel Sessions	16:30-18:10	See programme for Engineering 2	_	EN202	
Parallel Sessions	10.30-10.10	See programme for Mathematics, Computation & History	_	EN204		
			See programme for Design & Education	_	EN205	
	Pre-dinner Drinks	18:30-19:30	—	—	ATC103	
	Conference Banquet	19:30-23:00	_	_	Hawthorn Arts Centre	

Date	Event	Time	Details	Chair	Venue
	Registration	8:00-9:00	—	—	ATC103
	Plenary Lecture 5	9:00-9:50	Prof Glaucio H. Paulino (Princeton University, USA) Origami Engineering	Prof Larry	ATC101
	Plenary Lecture 6	9:50-10:40	Prof Zhong You (University of Oxford, UK) Beyond the Miura-ori: Innovations and Applications	Howell	ATOTOT
	Tea/Coffee Break	10:40-11:30	—	—	ATC103
			See programme for Engineering 1	—	EN201
	Parallel Sessions	11:30-12:30	See programme for Engineering 2	—	EN202
		11.50-12.50	See programme for Mathematics, Computation & History	—	EN204
			See programme for Design & Education	—	EN205
18 Jul	Lunch	12:30-14:00	_	—	ATC103
(Day 3)		See programme for Engineering 1	—	EN201	
	Parallel Sessions 14:00-16:00	See programme for Engineering 2	—	EN202	
	Parallel Sessions	14.00-10.00	See programme for Mathematics, Computation & History	_	EN204
			See programme for Design & Education (workshop: 45mins)	_	EN205
	Tea/Coffee Break	16:00-16:30	—	—	ATC103
			See programme for Engineering 1	_	EN201
	Parallel Sessions	16:30-17:30	See programme for Engineering 2	_	EN202
			See programme for Mathematics, Computation & History	_	EN204
			See programme for Design & Education	_	EN205
	Closing Ceremony	17:30-18:00	_	_	ATC101
	End of Conference				

Parallel Session I: Engineering 1 (Day 1)

Date: Tuesday, 16 July 2024 Time: 11:30-18:10 Venue: EN201

Time	Presenter	Title
	Rigid Origami (I) Chair: A/Prof Joe Gattas	
11:30-11:50	Danrong Shi, Jian Feng, Jin Zhang and Jianguo Cai (Southeast University, China)	Out-of-plane impact and energy absorption of origami honeycombs in Truck Mounted Attenuator
11:50-12:10	Changwoo Ha , Yasuhiro Miyazawa and Jinkyu Yang (Seoul National University, South Korea)	Quaternion-Based Loop Closure Method for Precise Kinematic Simulation of Rigid Origami
12:10-12:30	Kentaro Hayakawa and Makoto Ohsaki (Kyoto University, Japan)	Shape Optimization of Rigid Origami for Approximate Self-foldability under Gravity
12:30-14:00	Lunch (ATC103)	
	Rigid Origami (II) Chair: Prof Jianguo Cai	
14:00-14:20	Zeyuan He , Kentaro Hayakawa and Makoto Ohsaki (University of Cambridge, United Kingdom)	When will the existence of a non-trivial state guarantee a continuous motion for a quad-mesh rigid origami?
14:20-14:40	Rinki Imada , Thomas C. Hull, Jason S. Ku and Tomohiro Tachi (The University of Tokyo, Japan)	Nonlinear Kinematics of Recursive Origami Inspired by the Spidron
14:40-15:00	Collin Liou and Savannah Cofer (Stanford University, United States)	Programmable multistability and rigid flattenability in origami cubes by adding a minimal cut

	Rigid Origami (III) Chair: Prof Jiayao Ma	I
15:00-15:20	James P. McInerney, Xiaoming Mao, D. Zeb Rocklin, Glaucio H. Paulino and Diego Misseroni (University of Michigan, United States)	Isometries of trapezoid-based origami
15:20-15:40	Tomotaka Ohba , Kentaro Hayakawa and Makoto Ohsaki (Kyoto University, Japan)	Higher-order infinitesimal mechanism of rigid origami and polynomial approximation of its folding path
15:40-16:00	Alfonso Parra Rubio, Andy Dequin, Neil Gershenfeld and Erik Strand (Massachusetts Institute of Technology, United States)	Modular Origami Approach for Rigid Foldable Steel Load-Bearing Plate Lattices in Arbitrary Sizes
16:00-16:30	Tea/Coffee break (ATC1	03)
	Rigid Origami (IV) Chair: Dr Manan Arya	I
16:30-16:50	Jingsong Wei, Yan Chen and Xiao Zhang (Tianjin University, China)	Rigid-Foldable Hexagon-Twist Origami Patterns
16:50-17:10	Kiumars Sharifmoghaddam (The Vienna University of Technology, Austria)	T-hedral Origami: Towards Unifying Representation of Rigid-foldable and Curved Patterns
17:10-17:30	Klara Mundilova and Georg Nawratil (Massachusetts Institute of Technology, United States)	Rigid-Ruling Folding Compatibility of Planar Creases
17:30-17:50	Malcolm Smith (United States)	Amplifying the Kinematics of Origami Mechanisms Using Spring Joints
17:50-18:10	Kiumars Sharifmoghaddam , Klara Mundilova, Georg Nawratil and Tomohiro Tachi (The Vienna University of Technology, Austria)	Woven Rigidly Foldable T-hedral Tubes Along Translational Surfaces
	End of Day 1	

Parallel Session I: Engineering 1 (Day 2)

Date: Wednesday, 17 July 2024 Time: 11:30-18:10 Venue: EN201

Time	Presenter	Title	
	Thick Origami Chair: Prof Jiayao Ma		
11:30-11:50	David Bershadsky and Glaucio Paulino (Princeton University, United States)	Framework for the fabrication of flat foldable, thick origami structures via non-rigid origami methods	
11:50-12:10	Nachat Jatusripitak and Manan Arya (Stanford University, United States)	Regular and Semi-Regular Tessellations of Origami Flashers	
12:10-12:30	Hunter Pruett, Spencer Magleby and Larry Howell (Brigham Young University, United States)	Constant-Thickness Accommodation by Pattern Modification for Origami Flashers	
12:30-14:00	Lunch (ATC103)		
Curved Origami (I) Chair: A/Prof Jaehyung Ju			
14:00-14:20	Munkyun Lee , Mahmoud Abu-Saleem, Tomohiro Tachi and Joe Gattas (The University of Tokyo, Japan)	A lightweight building construction system using curved- crease origami blocks	
14:20-14:40	Ting-Uei Lee, Hongjia Lu and Yi Min Xie (RMIT University, Australia)	Designing Curved Folded Structures through Topology Optimisation	
14:40-15:00	Rupert Maleczek, Clemens Preisinger, Georg Lobe, Roland Maderebner and Marcus Bernhard (University of Innsbruck, Austria)	Actuating tubes in multilayer curved folding	

	Curved Origami (II) Chair: Prof Yoshinobu Miyan	noto
15:00-15:20	Rupert Maleczek , Klara Mundilova, Seri Nishimoto , Tomohiro Tachi and Riccardo Foschi (University of Innsbruck, Austria; The University of Tokyo, Japan)	Slit folding – openings to close along curved foldlines
15:20-15:40	Yiwei Zhang and Tomohiro Tachi (The University of Tokyo, Japan)	Rigid-ruling Curved Folding Origami Implemented with Straight Inflated Air Pouches
16:00-16:30	Tea/Coffee break (ATC10	3)
	Engineering Other (VI) Chair: A/Prof Joe Gattas	
16:30-16:50	Kai Xiao, Yuhao Wang, Chao Song, Bihui Zou, Zihe Liang, Heeseung Han, Yilin Du, Shane Johnson, Hanqing Jiang and Jaehyung Ju (Shanghai Jiao Tong University, China)	Topologically Variable and Volumetric Morphing of 3D Modular Origami Structures
16:50-17:10	Hongjia Lu, Ting-Uei Lee and Yi Min Xie (RMIT University, Australia)	Optimisation-based analysis of elastic buckling in cylinders with pre-embedded origami patterns
17:10-17:30	Phanisri Pratapa and Siva Poornan Vasudevan (Indian Institute of Technology Madras, India)	Finding the effective Poisson's ratios in stretching and bending of a reconfigurable morph origami
17:30-17:50	Mitch Skinner, Andrew Geyser, Barry Creighton, Collin Ynchausti, Spencer Magleby and Larry Howell (Brigham Young University, United States)	Cyclic Testing of Membrane Hinges for use in Origami- inspired Engineering Design
17:50-18:10	Ying Yu and Yi Wang (Shantou University, China)	Motion analysis of Flexible Modular Origami: A Finite Particle Method Investigation
	End of Day 2	

Parallel Session I: Engineering 1 (Day 3)

Date: Thursday, 18 July 2024 Time: 11:30-18:00 Venue: EN201

Time	Presenter	Title	
Kirigami (I) Chair: A/Prof Xiang Zhou			
11:30-11:50	Markus Holste , Kirsi Peltonen, Marcelo Dias and Leo de Waal (Aalto University, Finland)	Kirigami-inspired rectangular iso-area twist tessellations in architecture	
11:50-12:10	Keishi Kawahara, Nagi Nakamura, Satoshi Ikezawa , Kazuya Saito and Eiji Iwase (Waseda University, Japan)	Fabrication of DCRA using Kiri-origami Structure	
12:10-12:30	Jiahe Huang, Tuo Zhao, Glaucio Paulino and Yuhang Hu (Georgia Institute of Technology, United States)	Growing kirigami with self-healing and reprogrammable mechanical properties	
12:30-14:00	Lunch (ATC103)		
Kirigami (II) Chair: A/Prof Xinmei Xiang			
14:00-14:20	Shuangbo Liu , Jian Feng, Jianguo Cai and Qian Zhang (Southeast University, China)	A kirigami-inspired folding configuration for Muira thick panel	
14:20-14:40	Yoshinobu Miyamoto (Aichi Institute of Technology, Japan)	Rotational Erection System (RES) variations: fractals, tessellation, and interlinkage	
14:40-15:00	Kodai Nakagawa , Hibiki Totsuka, Miyako Mizuna, Nagi Nakamura, Tomohiro Tachi and Eiji Iwase (Waseda University Iwaselab, Japan)	Folding Condition of Kirigami and Rigid-foldable Kiri- origami Structure with Periodic Incision on Concentric Circles	

	Kirigami (III) Chair: Prof Ying Yu	
15:00-15:20	Miia Palmu , Kirsi Peltonen, Marcelo Dias, Leo de Waal and Tomi Kankkunen (VTT Technical Research Centre of Finland, Finland)	Design of morphing and multifunctional shape profiles through cutting tessellations
15:20-15:40	Chisaki Kitajima, Kazuya Saito and Kaoru Suehiro (Kyushu University, Japan)	Indusium Deployment of a Dictyophora Indusiata
15:40-16:00	Xinmei Xiang , Yingjing Liang, Jing Sun and Jingjing Wang (Guangzhou University, China)	Mechanical Properties of Gradient Miura-ori Metamaterials
16:00-16:30	Tea/Coffee break (ATC1)	03)
17:30-18:00	Closing Ceremony	
	End of Day 3	

Parallel Session II: Engineering 2 (Day 1)

Date: Tuesday, 16 July 2024 Time: 11:30-18:10 Venue: EN202

Time	Presenter	Title	
	Engineering Other (I) Chair: A/Prof Sachiko Ishi	da	
11:30-11:50	Tomohiro Arimune and Yuki Kobayashi (Osaka Metropolitan University, Japan)	A Flat Foldable Solid Consisting of Rhombitruncated Cuboctahedra and Regular Octagonal Prisms	
11:50-12:10	Yihe Wang , Wenze Zhang and Kai Tang (Hong Kong University of Science and Technology, China)	Kinematic Modeling of Cylindrical Origami Tessellations for Programmable Local Motion Control	
12:10-12:30	Yves Klett and Marc Grzeschik (Foldcore GmbH, Germany)	Fahrenheit 1832: Folding for Fire Protection	
12:30-14:00	Lunch (ATC103)		
Engineering Other (II) Chair: A/Prof Jiangmei Wu			
14:00-14:20	Mi Li , Huijuan Feng and Jian S. Dai (Southern University of Science and Technology/Tianjin University, China)	Topological manifold based parametric design of chiral origami mechanisms	
14:20-14:40	Jingyi Yang and Zhong You (University of Oxford, United Kingdom)	From Flexagon to Flexahedron – Infinitely Turning Objects	
14:40-15:00	Sicong Liu , Changjian Shen and Guoxing Lu (Southern University of Science and Technology, China)	Theoretical Analysis on the Deformation of the Miura- Ori Patterned Sheet	

Engineering Other (III) Chair: A/Prof Chao Chen			
15:00-15:20	Feng Qiao , Hong Xiao, Hongwei Guo, Rongqiang Liu and Zongquan Deng (Harbin Institute of Technology, China)	Deformable Origami Structure Design Based on Two- Dimensional Geometric Face Shape Collocation	
15:20-15:40	Zihang Ma, Keyao Song and Xiang Zhou (Shanghai Jiao Tong University, China)	Parametric Study of the Porous Origami-based Mechanical Metamaterials with Curvatures	
15:40-16:00	Kazuya Saito , Chisaki Kitajima, Kouki Nishi and Jun Sato (Kyushu University, Japan)	Earwig Fan Inspired Deployable Structures	
16:00-16:30	Tea/Coffee break (ATC10	3)	
Metamaterials Chair: Dr Jingyi Yang			
16:30-16:50	Akito Adachi, Seri Nishimoto , Hibiki Totsuka, Kanata Warisaya , Asao Tokolo and Tomohiro Tachi (The University of Tokyo, Japan)	Origami Cellular Material Switching Between Single and Multiple DOF Modes	
16:50-17:10	Savannah Cofer and Collin Liou (Stanford University, United States)	A flat-foldable, transformable metamaterial from octahedral origami unit cells	
17:10-17:30	Kevin T. Liu , Tuo Zhao and Glaucio H. Paulino (Princeton University, United States)	Kresling-Inspired Constant Size Magnetically- Reconfigurable Metamaterials	
17:30-17:50	Sachiko Ishida , Xinyi Zhang, Guoxing Lu and Kohei Okayasu (Meiji University, Japan)	Design and Fabrication of Quasi-isotropic Origami Metamaterials	
17:50-18:10	Kaili Xi , Xiaoyi Jiang, Jiayao Ma, Chuhan Xu and Yan Chen (Tianjin University, China)	Reconfigurable Mechanical Logic Module	
	End of Day 1		

Parallel Session II: Engineering 2 (Day 2)

Date: Wednesday, 17 July 2024 Time: 11:30-17:50 Venue: EN202

Time	Presenter	Title
	Engineering Other (IV) Chair: Prof Yusuke Maeda	a
11:30-11:50	Matthew Gardiner (Ars Electronica Futurelab, Austria)	Adding more bite to the origami chomper
11:50-12:10	Tomoko Taniguchi and Ryuhei Uehara (Japan Advanced Institute of Science and Technology, Japan)	Research on construction of double cubic core and its application
12:10-12:30	Jingyi Yang and Zhong You (University of Oxford, United Kingdom)	Kinematics analysis of Rubik's Magic puzzle and beyond
12:30-14:00	Lunch (ATC103)	
	Engineering Other (V) Chair: A/Prof Sicong Liu	
14:00-14:20	Nupur Udipi , Tuo Zhao and Glaucio Paulino (Georgia Tech, United States; Princeton University, United States)	New Kresling Origami Geometry: The Offset Cell
14:20-14:40	Matthew Gardiner , Anna Oelsch, Simon Schmid, Alexandre Bezri, Dan Wilcox, Anne Wichmann and Miller Puckette (Ars Electronica Futurelab, Austria)	Making origami musical instruments
14:40-15:00	Matthias Weber and Jiangmei Wu (Indiana University, United States)	Geometric Constructions of Bifoldable Polyhedral Complexes

	Robotics (I) Chair: A/Prof Cynthia Sur	ng
15:00-15:20	Matthew Gardiner, Simon Schmid, Anna Oelsch and Alexandre Bezri (Ars Electronica Futurelab, Austria)	Fold Sensing origami gestures - a case study with Kresling kinematics
15:20-15:40	Kevin Moreno Gata , Alex Seiter, Juan Musto, Judith Merz, Franziska Wieja, Georg Jacobs, Burkhard Corves and Martin Trautz (RWTH-Aachen University, Germany)	Design and development of a foldable and transformable hemispherical enclosure for robotic manufacturing
15:40-16:00	Jiang Lin , Lizi Deng, Huijuan Feng and Jian S. Dai (Southern University of Science and Technology, China)	Chiral origami robot with wheeled and quadcopter modes
16:00-16:30	Tea/Coffee break (ATC103	3)
	Robotics (II) Chair: Matthew Gardine	r
16:30-16:50	Yusuke Maeda , Shoma Sugisawa and Akitoshi Sakata (Yokohama National University, Japan)	A robotic origami folder for paper cranes
16:50-17:10	Duy Phuong Nguyen, Daphne Barretto , Darren Chiu, Elizabeth Cotter, Tuo Zhao, Jaime Fernandez Fisac and Glaucio H. Paulino (Princeton University, United States)	Miura-Bot: Modular Origami Robots for Self-Folding Miura-Ori Tessellations
17:10-17:30	Megan Ochalek, Manan Arya and Alexandra Haraszti (Stanford University, United States)	Adaptive Stiffness and Shape Control of a Modular Origami-Inspired Robot
17:30-17:50	Gabriel Unger and Cynthia Sung (University of Pennsylvania, United States)	Re-programmable Matter by Folding: Magnetically- Controlled Origami that Self-Folds, Self-Unfolds, and Self-Reconfigures On-Demand
	End of Day 2	

Parallel Session II: Engineering 2 (Day 3)

Date: Thursday, 18 July 2024 Time: 11:30-18:00 Venue: EN202

Time	Presenter	Title	
Engineering Other (VI) Chair: A/Prof Jiangmei Wu			
11:30-11:50	Simon Thissen, Yves Klett and Peter Middendorf (Foldcore GmbH, Germany)	Tessellation Manufacture by Sequential Quasi-Isometric Gradual Folding	
11:50-12:10	Tuo Zhao and Glaucio Paulino (Princeton University, United States)	Programming Origami Instabilities via Topology Optimization	
12:10-12:30	Qian Zhang, Jian Feng and Jianguo Cai (Southeast University, China)	Cut design of pop-up origami with fixed planar substrate	
12:30-14:00	Lunch (ATC103)		
Engineering Other (VII) Chair: Dr Jianjun Zhang			
14:00-14:20	Kaori Kuribayashi-Shigetomi , Takashi Horiyama and Ryuhei Uehara (Hokkaido University, Japan; Japan Advanced Institute of Science and Technologies, Japan)	3D tissues using truncated octahedron blocks produced by Origami and micro/nano processing technologies	
14:20-14:40	Mao Qin, Qiuyue Zhong, Runmin Qian, Qian Zhang and Jianguo Cai (Southeast University, China)	Integrated non-destructive development design of oblique cutting four pyramid frustum origami shading cover	
14:40-15:00	Ke Liu and Glaucio Paulino (Peking University, China)	Symmetric self-folding of N-gon hypar origami	
16:00-16:30	Tea/Coffee break (ATC103)		
17:30-18:00	Closing Ceremony		
End of Day 3			

Parallel Session III: Mathematics, Computation & History (Day 1)

Date: Tuesday, 16 July 2024 Time: 11:30-17:50 Venue: EN204

Time	Presenter	Title
	Mathematics (I) Chair: Miyuki Kawamura	
11:30-11:50	Michael Assis (University of Melbourne, Australia)	Folding Pi
11:50-12:10	Erik D. Demaine, Robert Lang, Klara Mundilova and Tomohiro Tachi (Massachusetts Institute of Technology, United States)	Analysis of Huffman's Hexagonal Column with Cusps
12:10-12:30	Tonan Kamata , Takumi Shiota and Ryuhei Uehara (Japan Advanced Institute of Science and Technology, Japan)	A Characterization of the Overlap-free Polyhedra
12:30-14:00	Lunch (ATC103)	
	Mathematics (II) Chair: Ryuhei Uehara	
14:00-14:20	Kazuki Matsubara and Chie Nara (Saitama University, Japan)	Continuous flattening of quadrangular prisms with all edges rigid except one
14:20-14:40	Chie Nara (Meiji university, Japan)	Continuous flattening of the surface of regular star- polyhedra
14:40-15:00	Manami Niijima (JVCKENWOOD Corporation, Japan)	On Beloch's curve that appears when solving real cubics with origami

Mathematics (III) Chair: A/Prof Kaori Shigetomi		
15:00-15:20	Travis Nolan (Southern Methodist University, United States)	22.5-degree References via Continued Fractions
15:20-15:40	Ryuhei Uehara (Japan Advanced Institute of Science and Technology, Japan)	A survey of rep-cube
15:40-16:00	Yoshikazu Yamagishi (Ryukoku University, Japan)	Star unfolding, source unfolding, and the farthest point mapping on the 4-cube
16:00-16:30	Tea/Coffee break (ATC1)	03)
	Mathematics (IV) Chair: Dr Mike Assis	
16:30-16:50	Tianhao Zhang and Ken'Ichi Kawaguchi (Meijo University, Japan)	Preliminary research on the bending behavior of curved origami in shape determination
16:50-17:10	Robert Geretschläger (BRG Kepler, Austria)	Finding Solutions to Origami Boundary Value Problems with Edges of Platonic Solids
17:10-17:30	Hussein Nassar and Andrew Weber (University of Missouri, United States)	Asymptotically-isometric deformations of periodic piecewise-smooth surfaces
17:30-17:50	Hiroko Murai (Nara Women's University, Japan)	Some applications of topology on origami
	End of Day 1	

Parallel Session III: Mathematics, Computation & History (Day 2)

Date: Wednesday, 17 July 2024 Time: 11:30-18:10 Venue: EN204

Time	Presenter	Title		
	Computation (I) Chair: Prof Tomohiro Tachi			
11:30-11:50	Hugo Akitaya, Erik D. Demaine and Jason S. Ku (National University of Singapore, Singapore)	Computing Flat-Folded States		
11:50-12:10	Michael Assis (University of Melbourne, Australia)	An origami Universal Turing machine design		
12:10-12:30	Wojtek Burczyk (Artist, Portugal)	A Layering Algorithm for Flat Modular Origami		
12:30-14:00	Lunch (ATC103)			
	Computation (II) Chair: Prof Jun Mitani			
14:00-14:20	Lily Chung, Erik D. Demaine , Martin Demaine, Jenny Diomidova, Jayson Lynch, Klara Mundilova and Hanyu Zhang (Massachusetts Institute of Technology, United States)	Folding a Strip of Paper into Shapes with Specified Thickness		
14:20-14:40	Daniel Feshbach , Wei-Hsi Chen, Daniel Koditschek and Cynthia Sung (University of Pennsylvania, United States)	Kinegami: Open-source Software for Creating Kinematic Chains from Tubular Origami		
14:40-15:00	Wessel van der Heijden, Irina Kostitsyna and Jason S. Ku (National University of Singapore, Singapore)	Equilateral triangle map folding		

	Computation (III) Chair: Dr Mike Assis	
15:00-15:20	Yiyang Jia and Jun Mitani (Seikei University, Japan; University of Tsukuba, Japan)	Valid Boundary Orders in Principal Diagonal Grid Patterns via Simple Folds
15:20-15:40	Jason S. Ku , Akira Terao and Kenji N. Terao (National University of Singapore, Singapore)	An Algebraic Approach to Layer Ordering Constraints for Origami Flat-Foldability
15:40-16:00	Yusuke Maeda , Hiroki Tabata, Naruya Suzuki and Yuji Nakajima (Yokohama National University, Japan)	An Origami Simulator for Papers with Nonzero Thickness and Its Application to Support Folding Nonelementary Origami Works
16:00-16:30	Tea/Coffee break (ATC103)
Computation (IV) Chair: Dr Robert Lang		
16:30-16:50	Chihiro Nakajima (Tohoku Bunka Gakuen University, Japan)	An Efficient Enumeration of Flat-Foldings: Study on Random Single Vertex Origami
16:50-17:10	Koji Ouchi, Hideo Komatsu and Ryuhei Uehara (Japan Advanced Institute of Science and Technology, Japan)	Efficient enumeration of rectangles in origami design
17:10-17:30	Sora Seshima and Jun Mitani (University of Tsukuba, Japan)	Shape Modeling of Developable Surfaces with a Curve Crease by Extending the Handle Curve Approach
17:30-17:50	Ryuhei Uehara (Japan Advanced Institute of Science and Technologies, Japan)	Optimal Simple Fold-and-Cut of a Polygonal Line
17:50-18:10	Brandon Wong and Erik D. Demaine (Massachusetts Institute of Technology, United States)	Algorithmic transitions between parallel pleats
	End of Day 2	

Parallel Session III: Mathematics, Computation & History (Day 3)

Date: Thursday, 18 July 2024 Time: 11:30-18:00 Venue: EN204

Time	Presenter	Title	
	History		
	Chair: Clare Chamberlain		
11:30-11:50	Koshiro Hatori	Pseudohistories of the origin of origami in Japan and	
	(Japan Origami Academic Society, Japan)	Europe	
11:50-12:10	Eiko Matsuura	"Origami" as a Universal Term: Akira Yoshizawa's	
	(Japan Origami Academic Society, Japan)	Perspective	
12:10-12:30	Adam Woodhouse and Joseph Munro	Decision making in an Origami Archive – A story of the	
	(British Origami Society / University of Essex, United Kingdom)	British Origami Society Library	
12:30-14:00	Lunch (ATC103)		
	Demonstration (I)		
	Chair: Winnie Leung		
14:00-15:00	Jun Maekawa	Making some geometric models	
	(Origami artist, Japan Origami Academic Society, Japan)	Making some geometric models	
	Demonstration (II)		
	Chair: Sukanya Deshmukh		
15:00-15:30	Matthew Elvey Price	Interlocking modular origami (wireframe) design with	
	(Independent, Australia)	geometry creation and optimisation tools	
15:30-16:00	Koya Narumi and Tomohiro Tachi	Inkjet 4D print	
	(Keio University, Japan)		
16:00-16:30	Tea/Coffee break (ATC103)		

Demonstration (III) Chair: Sukanya Deshmukh			
16:30-17:30	Raymond Feng (Monta Vista High School)		Principles and Applications of Kusudama Design
17:30-18:00		Closing Ceremony	
End of Day 3			

Parallel Session IV: Design & Education (Day 1)

Date: Tuesday, 16 July 2024 Time: 11:30-17:30 Venue: EN205

Time	Presenter	Title
	Design Theory Chair: Dr Jason Ku	
11:30-11:50	Brandon Wong (MIT, United States)	New techniques in hex pleating for representational origami design
11:50-12:10	Madonna Yoder (Gathering Folds, United States)	Symmetry Requirements and Design Equations for Origami Tessellations
12:10-12:30	Mu-Tsun Tsai and Robert J. Lang (langorigami.com, United States)	On the Constructions of Generalized Offset Pythagorean Stretch Patterns
12:30-14:00	Lunch (ATC103)	
	Tessellations (I) Chair: Winnie Leung	
14:00-14:20	Sukanya Deshmukh and Michael Assis (RMIT University, Australia; University of Melbourne, Australia)	Exploring criteria for designing novel waterbomb tessellations using triangular convex polygons
14:20-14:40	Mamoru Doi (Independent Researcher, Japan)	Flat-back 3D gadgets in origami extrusions completely downward compatible with the conventional pyramid-supported 3D gadgets
14:40-15:00	Madonna Yoder (Gathering Folds, United States)	Hybrid Hexagon Twist Interface

		Tessellations (II) Chair: Benjamin Parker	
15:00-15:20	Tetsuya Yoshida (Nara Women's University, Japan)	-	Generating Smocking Patterns of Twist Folds for Clothing Design
15:20-15:40	Tetsuya Yoshida and Misa Tada (Nara Women's University, Japan)		Twist Fold Modules for Combinatorial Design of Petaloid Smocking in Clothing
15:40-16:00	Tetsuya Yoshida and Ayako Nakanishi (Nara Women's University, Japan)		Visualizing Petaloid Smocking based on Rotation of Decorations and Pleat Length
16:00-16:30	Т	rea/Coffee break (ATC103)	
Tessellations (III) Chair: Madonna Yoder			
16:30-16:50	Yohei Yamamoto and Jun Mitani (University of Tsukuba, Japan)		Comparing Twist Pattern Design Method and Design Methods of Primal-Dual Tessellations
16:50-17:10	Jiangmei Wu (Indiana University, United States)		Fabric Origami Tessellation: From Sewing Patterns to Crease Patterns
17:10-17:30	Benjamin DiLeonardo-Parker (Ben Parker Studio, United States)		A Systematic Notation to Pleat Intersection Operations
		End of Day 1	

Parallel Session IV: Design & Education (Day 2)

Date: Wednesday, 17 July 2024 Time: 11:30-18:10 Venue: EN205

Time	Presenter	Title
	Education (I)	
	Chair: Thais Regina Ueno Ya	amada
11:30-11:50	Norma Boakes (Stockton University, United States)	Exploring participant perspectives through a collaborative art project in a university origami mathematics course
11:50-12:10	Carolina Graciolli (São Paulo State University (UNESP), Brazil)	Origami and Video Production in Mathematics Education
12:10-12:30	Carolina Graciolli and Franciele Teixeira (São Paulo State University (UNESP), Brazil)	Euclidean, Spherical and Origami Geometry: possibilities in Mathematics Education
12:30-14:00	Lunch (ATC103)	
	Education (II) Chair: Norma Boakes	
14:00-14:20	Rebecca Ottinger, Kristin Komatsubara, Perla Myers , Lisa Smith and Anna Walsh (University of San Diego, United States)	Enhancing Elementary Education: Enhancing the Teaching of Visuospatial Skills through Professional Learning and Implementation of a Unique Origami Curriculum
14:20-14:40	Tiina Kraav and Anne Rudanovski (University of Tartu, Estonia)	Educational Origami: The Design and Implementation of the Course 'Mathematical Paper Folding' in Upper Secondary School
14:40-15:00	Hisae Miyauchi (University of Tsukuba, Japan)	Exploring Origami's Impact: Nurturing Spatial Skills to Enhance STEM Access among Individuals with Blindness

	Education (III) Chair: Prof Perla Myers		
15:00-15:20	Laure Ninove (UCLouvain, Belgium)	Approaching proof in geometry by folding problems with pre-service middle school mathematics teachers	
15:20-15:40	Larissa S. Novelino and Glaucio H. Paulino (Rice University, United States; Princeton University, United States)	Simplifying the Fold-and-One-Cut Problem: A Pedagogical Approach for Origami Engineering Education	
15:40-16:00	Jacoliene van Wijk , Anna Shvarts, Bos Rogier and Michiel Doorman (Utrecht University, Netherlands)	Engaging secondary school students in building formulas based on mathematical folding	
16:00-16:30	Tea/Coffee break (ATC103)		
Education (IV) Chair: Laure Ninove			
16:30-16:50	Kaori Shigetomi (Hokkaido University, Japan)	Critical thinking class using Origami	
16:50-17:10	Thais Regina Ueno Yamada (São Paulo State University (UNESP), Brazil)	The use of cutting and folding techniques in the creative process by Arts and Product Design students	
17:10-17:30	Anna Walsh, Perla Myers , Kristin Komatsubara and Rebecca Ottinger (University of San Diego, United States)	Mathigami, 6OSME Documentary, and Learning Visual Spatial Teaching in 2023	
17:30-17:50	Perla Myers , Lisa Smith, David Geary, Alora Li, Rebecca Ottinger, Vitaliy Popov and Zehra Unal (University of San Diego, United States)	Origami and Mathematics in the classroom: Increasing Spatial Vocabulary and Decreasing Mathematics Anxiety through Paper Folding	
17:50-18:10	Robert Fletcher and Michael Assis (University of Melbourne, Australia)	Vision impaired origami teaching and folding - a Melbourne experience	
	End of Day 2		

Parallel Session IV: Design & Education (Day 3)

Date: Thursday, 18 July 2024 Time: 11:30-18:00 Venue: EN205

Time	Presenter	Title
	Design Other (I) Chair: Matthew Gardiner	
11:30-11:50	Daniel Brown and Jason S. Ku (National University of Singapore, Singapore)	Folding all 4 × 4 Rotationally-Symmetric Diagonal-Grid 2-Color Patterns
11:50-12:10	Wenwu Chang, Siheng Yu and Jinrui Zhang (PUTUO Institute of Modern Educational technology, China)	From A4 paper to Tangram Puzzles: The Math Behind the Paper Folding
12:10-12:30	Naddhawara Mahanukul, Stuart Favilla and Erica Tandori	Pop-up multisensory science books and STEM exhibition design for people living with low vision, blindness, and diverse needs.
12:30-14:00	Lunch (ATC103)	
	Design Other (II) Chair: Mike Assis	
14:00-14:20	Mamoru Doi (Independent Researcher, Japan)	Rotational origami of polyhedral type and reduction of flanges
14:20-14:40	Anne Rudanovski and Tiina Kraav (Pallas University of Applied Sciences, Estonia)	The multifaceted dialogue initiated by the origami- based artistic process
14:40-15:00	Jun Mitani (University of Tsukuba, Japan)	Pillow Box Design

Design Other (III) (Workshop: 45mins) Chair: Winnie Leung		
15:00-16:00	Goran Konjevod (organicorigami.com, United States)	Folding curves over pleats
16:00-16:30	Tea/Coffee break (ATC103)	
Design Other (IV) Chair: Goran Konjevod		
16:30-16:50	Stephen Lerangtes and Winnie Leung (Sydney Origami Inc, Australia)	Topological Transformation of the Miura Ori Crease Pattern
16:50-17:10	Adam Woodhouse, Abdellah Salhi and Joseph Munro (University of Essex, United Kingdom)	Exploring Collapsible Origami Structures for Beehives, Portable Toilets and Wheelchairs - Challenges and Opportunities in Design Innovation
17:10-17:30	Frances Winters (University of New England, Australia)	Folding investigations led to the design of multiplanar tessellations linked by circular 4 colouring permutations. Folding across this array of geometric coloured glyphs results in 3D enantiomorphic, non-numeric "dice"
17:30-18:00	Closing Ceremony	
End of Day 3		

List of Posters and Exhibits Venue: ATC103 Date: 16-18 July

Authors	Title
Tetsuya Yoshida (Nara Women's University, Japan)	Generating Strings from Crease Patterns for Facilitating the Folding of Petaloid Smocking
Mamoru Doi (Independent Researcher, Japan)	Triangle-supported negative 3D gadgets in origami extrusions with a canonical correspondence to flat-back positive 3D gadgets
Mamoru Doi (Independent Researcher, Japan)	Truncated 3D gadgets in origami extrusions
Mamoru Doi (Independent Researcher, Japan)	A variational approach to the paper bag problem for flanged origami packages folded from dihedrons of convex polygons
Ren Hashiguchi, Chisaki Kitajima and Kazuya Saito (Kyushu University, Japan)	Designing of a Novel Umbrella Based on the Bricard Linkage
Malcolm Smith (United States)	Amplifying the Kinematics of Origami Mechanisms Using Spring Joints
Gabriel Unger (University of Pennsylvania, United States)	Re-programmable Matter by Folding: Magnetically-Controlled Origami that Self-Folds, Self-Unfolds, and Self-Reconfigures On- Demand
Madonna Yoder (Gathering Folds, United States)	Origami Exhibits
Yohei Yamamoto (University of Tsukuba, Japan)	Origami Exhibits
Jun Mitani (University of Tsukuba, Japan)	Origami Exhibits

Jiangmei Wu (Indiana University, United States)	Origami Exhibits
David Bershadsky (Princeton University, United States)	Origami Exhibits
Alfonso Parra Rubio (Massachusetts Institute of Technology, United States)	Origami Exhibits
Markus Holste (Aalto University, Finland)	Origami Exhibits
Brandon Wong (Massachusetts Institute of Technology, United States)	Origami Exhibits
Miia Palmu (VTT Technical Research Centre of Finland, Finland)	Origami Exhibits
Akito Adachi, Seri Nishimoto, Kanata Warisaya and Tomohiro Tachi (The University of Tokyo, Japan)	Origami Exhibits
Darryl Bedford (Drawstring Origami, UK)	Origami Exhibits
Matthew Gardiner (Ars Electronica Futurelab, Austria)	Origami Exhibits
Malcolm Smith (United States)	Origami Exhibits
Nupur Udipi (Georgia Tech, United States)	Origami Exhibits
Tomohiro Arimune and Yuki Kobayashi (Osaka Metropolitan University, Japan)	Origami Exhibits

Rupert Maleczek (University of Innsbruck, Austria; The University of Tokyo, Japan)	Origami Exhibits
Yoshinobu Miyamoto (Aichi Institute of Technology, Japan)	Origami Exhibits

Notes

8th International Meeting on Origami in Science, Mathematics and Education (8OSME)

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Parallel Session I: Engineering 1

Out-of-plane impact and energy absorption of origami honeycombs in Truck Mounted Attenuator

Danrong Shi¹, Jian Feng¹, Jin Zhang¹, Jianguo Cai¹

¹ Southeast University, China

A Truck Mounted Attenuator (TMA) is a device mounted on road maintenance vehicles, positioned at the rear to establish a buffer zone between the vehicle and the construction work area ahead. A numerical model of the TMA, consisting of a steel skin, steel frame, and origami aluminum honeycomb, has been developed. This model can be extended to larger-scale collision scenarios, particularly those involving real vehicle models. The substantial cost-effectiveness of having a validated numerical model enables automotive crash cushion designers to perform extensive testing through numerical simulations prior to standard impact tests. The study also explored the energy-absorption characteristics of origami honeycomb during the out-of-plane crushing process within the TMA. Numerical simulation results indicate that origami honeycomb offers a more stable folding process compared to traditional honeycomb. The findings of this research will assist transportation agency designers in the development of the next generation of vehicle-mounted crash cushions, thereby enhancing the safety of highway maintenance vehicles.

Keywords: Origami honeycomb; Truck Mounted Attenuator (TMA); Impact; Energy absorption

Quaternion-Based Loop Closure Method for Precise Kinematic Simulation of Rigid Origami

Changwoo Ha¹, Yasuhiro Miyazawa¹, and Jinkyu Yang¹

¹ Seoul National University, South Korea

This study describes a novel approach utilizing quaternions for loop closure constraints in rigid origami structures. Our research outlines the modeling of quaternions for rotation and loop closure constraints, emphasizing their advantages over traditional matrix-based methods. We also present numerical solutions using the Newton-Raphson method for finding folding angles in origami patterns, comparing the performance of quaternion-based and matrix-based methods through examples such as the Miura-ori pattern and a 4-closed-vertex pattern. Results indicate the effectiveness of the quaternion approach, particularly in multi-vertex origami problems.

Keywords: Quaternion; Loop closure constraints; Rigid origami simulations

Shape Optimization of Rigid Origami for Approximate Selffoldability under Gravity

K. Hayakawa¹, M. Ohsaki¹

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We explore the methods to achieve an approximately self-foldable crease pattern of flat rigid origami whose motion is driven by gravity. The approximate self-foldability is attempted from the mechanics of the panel-pin model developed for the equilibrium analysis of rigid origami. The predominant deformation mode under gravity is estimated from the total potential energy and the eigenvalues of the stiffness matrix of the model that has small rotational stiffness along the crease lines. The arrangement of the crease lines is optimized by maximizing the gap between the eigenvalues or the total potential energy variation in the direction of the eigenvectors of the stiffness matrix.

Keywords: Rigid origami; Self-folding; Multiple-degree-of-freedom mechanism; Equilibrium; Gravity

When will the existence of a non-trivial state guarantee a continuous motion for a quad-mesh rigid origami?

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Regarding quad-mesh rigid origami, the flat-foldable pattern exhibits a widely acknowledged geometrical property – a non-trivial state guarantees a continuous motion. This article provides a sufficient condition for this motion-guarantee property in the context of a generalized quad-mesh rigid origami. Our method is a detailed algebraic analysis on the coupled two-vertex systems constituting a quad-mesh rigid origami. The result is expected to be a useful tool for the inverse design using quadmesh rigid origami.

Keywords: Rigid-foldable; V-hedra; T-hedra; Inverse design; Polyhedral surfaces; Discrete conjugate net; Isometric deformation

Nonlinear Kinematics of Recursive Origami Inspired by the Spidron

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Non-periodic folding of periodic crease patterns paves the way to novel nonlinear phenomena that cannot be feasible through periodic folding. This paper focuses on the non-periodic folding of recursive crease patterns generalized from Spidron. Although it is known that Spidron has a 1-DOF isotropic rigid folding motion, its general kinematics and dependence on the crease pattern remain unclear. Using the kinematics of a single unit cell of the Spidron and the recursive construction of the folded state of multiple unit cells, we consider the folding of the Spidron that is not necessarily isotropic. We found that as the number of unit cells increases, the non-periodic folding is restricted and the isotropic folding becomes dominant. Then, we analyze the three kinds of isotropic folding modes by constructing 1-dimensional dynamical systems governing each of them. We show that these systems can possess different recursive natures depending on folding modes even in an identical crease pattern. Furthermore, we show their novel nonlinear nature, including the period-doubling cascade leading to the emergence of chaos.

Keywords: Rigid Origami; Dynamical System; Bifurcation; Chaos

Programmable multistability and rigid flattenability in origami cubes by adding a minimal cut

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Rigid origami is a model of origami in which thin, rigid panels are joined by hinges that allow the surface to fold. Understanding rigid folding behavior is important for a variety of engineering applications, especially in the context of deployable origami mechanisms fabricated from high modulus materials like acrylic, metal, and cardboard. While convex polyhedra constructed of rigid faces and hinged edges are themselves rigid even if a finite number of creases are added to the faces, introducing a small cut to the surface of the polyhedron eliminates rigidity and may allow for a polyhedron to be completely flattened. In particular, we are interested in the development of a minimally altered, rigidly flattenable origami cube for applications in deployable structures and shape-changing robots. In this work, we present a kinematic model of a bistable, flat foldable origami cube. We also implement an analytic framework to minimize the cut length that allows for a rigid flattening of this cube, finding the minimum to be four identical cuts, each of which are approximately 8.6% of the side length. This approach was experimentally verified with tensile testing to construct an energy profile of the cube over deformation. Additionally, by changing the number of cuts that are placed on the cube, we can predictably manipulate or eliminate the energetic barrier when flattening the cube, as well as adjust the amount of energy dissipated over a deformation cycle. Overall, we experimentally demonstrate the effect of adding small slits on the system's energetic behavior to enable rigid foldability and programmable stiffness, thus opening up the possibility for usage in engineering and robotic applications.

Keywords: Programmable multistability; Rigid origami; Flat foldability; Origami cube; Minimum cut

Isometries of trapezoid-based origami

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Rigidly foldable origami tessellations require a careful balance between folding degrees of freedom and geometric compatibility constraints. However, such tessellations generically exhibit non-rigid linear isometries that allow the panels to bend without stretching. While these linear isometries can be modeled via a triangulation of the crease pattern, it is challenging to analytically characterize the deformations in all but the simplest geometries. Here, we discuss a recently-developed theoretical framework for characterizing the linear isometries of quadrilateral-based origami tessellations that more clearly distinguishes between folding and bending degrees of freedom. We showcase the utility of this framework using examples of trapezoid-based origami (including the special case of parallelograms), from which we deduce capabilities of such crease patterns as mechanical metamaterials.

Keywords: Origami; Rigid folding; Isometries

Higher-order Infinitesimal Mechanism of Rigid Origami and Polynomial Approximation of Its Folding Path

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We propose a systematic scheme for computing the second- and higher-order infinitesimal mechanisms of a rigid origami modeled as a truss structure for approximating its folding path by a polynomial obtained from the series expansion of the nodal displacement with respect to the path parameter controlling the folding state. The first- to n-th-order terms of the nodal displacement are determined so that the firstto n-th-order terms of the series expansion of the compatibility equations are to be satisfied with identically zero. These terms can be systematically computed from the first-order infinitesimal mechanism satisfying the existence conditions of the infinitesimal mechanisms up to n-th-order. The approximation accuracy of the finite folding path is verified by varying the order of terms considered in the approximation.

Keywords: Rigid origami; Finite fold; Infinitesimal mechanism; Truss model; Triangular face

Modular Origami Approach for Rigid Foldable Steel Load-Bearing Plate Lattices in Arbitrary Sizes

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While plate lattices exhibit superior mechanical performance compared to truss lattices at equivalent densities, their fabrication on an engineering scale presents significant challenges, particularly when utilizing structural materials for load-bearing applications. Industry and academia predominantly rely on 3D printing techniques, but encounter limitations in feature size and scalability of energy efficiency for high production rates. In this study, we propose a modular origami design and manufacturing method for creating steel plate lattices at the meso-scale using sheet stock. Rigid-foldable unit cells are cut, folded, and discretely assembled into lattices. Their mechanical performance is simulated and mechanically tested.

Keywords: Metamaterials; Plate Lattices; Structural Origami; Metallic Origami

Rigid-Foldable Hexagon-Twist Origami Patterns

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Flat-twist origami patterns are of superiorly foldable properties, therefore, have been used to design metamaterials and deployable structures. However, most of the flattwists such as hexagon-twist patterns are non-rigid foldable which limits their applications due to the complex deformations. Here, several rigid-foldable hexagontwist patterns with lower degree-of-freedom (DOF) are proposed by adding creases and modifying the mountain-valley arrangements. Four one-DOF two-dimensional tessellations of rigid hexagon-twist patterns are created by translationally connecting facets and creases. Furthermore, one of four tessellations is chosen to construct a threedimensional tessellation by sharing specific edges for the potential design of metamaterial. The work widens the road to construct rigid-foldable origami patterns.

Keywords: Rigid foldability; Hexagon-twist origami; Tessellation; Lower degree-of-freedom

T-hedral Origami: Towards Unifying Representation of Rigid-foldable and Curved Patterns

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A T-hedron is a flexible planar quad mesh with combinatorics of a regular grid composed of rigid trapezoidal panels and rotational hinges. Recent studies on discrete and semi-discrete analogues of these surfaces, uncovered explicit construction and class-preserving isometric deformation of their one-degree of-freedom movement. Building upon this concept, T-hedra is employed as a novel representation of wellknown origami patterns and non-developable as well as curve-folded tessellations. This study demostrates that many rigid-foldable quadrilateral patterns with one family of straight creases, including Miura-ori, chicken wire, eggbox and their variations, fall within this classification. Furthermore, considering degenerate cases, T-hedra can represent patterns with other combinatorics, such as Yoshimora, Kresling and waterbomb and their class-preserving isometric deformation which yields analytic folding motion. This representation, enables creation of generalized and hybrid versions all encapsulated within a single T-hedron. Additionally, T-hedra introduces a novel family of non-developable yet rigid- and flat-foldable structures, allowing crossed quadrilaterals which can be physically realized, exhibiting intriguing kinematic movements. To bolster the theoretical framework and demonstrate the expanded design potential for rigid and curved origami tessellations, an interactive plugin for Grasshopper/Rhino has been developed. This tool empowers users to design such patterns in any folded configuration and visualize their precise geometric deformation in real time.

Keywords: Rigid-foldable; Curved folding; Flexible quad surfaces; Geometric classification; Non-Euclidean Origami

Rigid-Ruling Folding Compatibility of Planar Creases

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Planar curved creases represent an important family of creases and offer significant potential for application in interactive design tools. Once a folded state is constructed, the question arises: Does a folding motion exist, maintaining the same ruling layout, that connects the flat state to the folded state? This is referred to as a rigid-ruling folding motion. In this paper, we characterize combinations of planar curves that allow a rigid-ruling folding motion and illustrate our theoretical findings with examples.

Keywords: Curved-crease origami; Planar creases; Rigid-ruling folding motion

Amplifying the kinematics of origami mechanisms with spring joints

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Due to its rigid foldability and predictable kinematics, the reverse fold is the fundamental mechanism behind some of the most well known origami kinematic structures, including the Miura Ori, Yoshimura, and waterbomb patterns. However, the reverse fold only has one parameter to control its behavior: the starting fold angle. In this paper I introduce an alternative to the traditional reverse fold-based on the springinto-action pattern-called the spring joint. This novel rigidly foldable mechanism is able to couple multiple reverse folds into a compact space to amplify the kinematic output of a traditional reverse fold by up to ten times, and to add one parameter for each reverse fold, giving more programmatic control of origami structures. Methods of parameterizing both the starting angle, the path of travel, and the axis of motion are also introduced such that the spring joint can be engineered to any application within compliant mechanisms, deployable structures and robotics. Unfortunately, this versatility comes at the cost of a large buildup of layers, making the spring joint impractical for thick origami mechanisms. To solve this problem, I also introduce a modular alternative to the spring joint that has no additional layers, with the same kinematic properties. Both of these mechanisms are tested as replacements for the reverse fold in both traditional and custom origami structures.

Keywords: Origami; Mechanism; Deployable Structure; Rigidly foldable

Woven Rigidly Foldable T-hedral Tubes Along Translational Surfaces

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Rigidly foldable tubes, initially derived from mirroring and interconnecting corresponding edges of Miura-ori cells, possess bi-directional flat-foldability and exhibit out-of-plane stiffness due to their one-degree-of-freedom kinematic movement in their regular configuration. Various methods have been established for enhancing their structural complexity and stiffness, including edge and face connections, as well as zipping and interleaving, leading to the creation of advanced structures like foldable sandwiches and metamaterials.

In this study, we introduce a novel family of cellular structures where orthogonally aligned rigid-foldable tubes are in a woven configuration. We show a computational process to achieve the tubular designs following a translational surface for any given weaving pattern (including existing interleaved tubes) by introducing the idea of flip-flop joints that can switch the up-and-down relation between contact tubes by changing their orientation.

This research not only expands the repertoire of origami-based engineering techniques but also offers a practical toolset for designers and engineers seeking innovative solutions in structural optimization.

Keywords: Rigid-foldable; Origami tubes; Cellular structures; Metamaterials; Weaving patterns; Discrete translational surface

Framework for the fabrication of scalable, flat foldable, thick origami Kresling structures via non-rigid methods

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The fabrication of thick origami structures often faces two challenges. Thick origami is usually rigid and not flat foldable. This work demonstrates a technique for creating nonrigid, fully flat foldable, and thick origami. By harnessing the material properties of 3D-printed Polylactic Acid (PLA), thick origami panels that selectively exhibit nonrigid behaviors can be fabricated using living hinges. Constraining the deformation of these living hinges within the elastic regime of the material, robust flat foldable thick origami structures can be created. These methods are demonstrated in a parametric model of a thick origami flat foldable Kresling pattern. The kinematics and bistable mechanical properties of these Kresling structures will be characterized to demonstrate the viability and scalability of this technique.

Keywords: Origami; Kresling; Flat Foldable; Thick Origami; Non Rigid Origami

Regular and Semi-Regular Tessellations of Origami Flashers

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Origami wrapping fold patterns, also known as flashers, allow for the n-fold rotationally symmetric wrapping of a sheet around a central polygon. We show how origami flashers may tessellate the plane through regular and semi-regular tilings. Crucially, both the folded and unfolded forms can be shown to tessellate. This allows for the entire tessellation to fold regardless of the number of unit cells. This work has the potential to impact the design of modular shape-changing structures. We present conditions on zero-thickness flasher unit cells that allow for tessellation via regular and semi-regular tilings. Each such tessellation corresponds to a family of flasher-based fold patterns. Additionally, we present an algorithm to generate thickness-accommodating spiral-wrapped flasher unit cells that can tessellate.

Keywords: Wrapping patterns; Flashers; Tessellation; Thickness accommodation

Constant Thickness Accommodation for Origami Flashers

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Thickness accommodation is one of the most significant challenges in creating an origami flasher array. If it is a membrane array, thickness accommodation is generally simpler as panel deformation allows the array to wrap about itself with relative ease. However, for rigid-panel arrays, thickness accommodation is considerably more complicated. This work develops thickness accommodation techniques that yield origami-inspired mechanisms of constant thickness and high-volume efficiency. In this paper, current methods for accommodating thickness in flashers are presented and issues associated with those methods are discussed. Two methods for accommodating thickness are proposed.

Keywords: Thickness accommodation; Origami flasher; Deployable space systems

A lightweight building construction system using curvedcrease origami blocks

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This paper proposes a novel modular building system using curved-crease origami columns. The column is designed with a non-rotationally symmetric trapezoidal cross-section profile and an elastica curvature surface profile. The selected profile and surface curvature allows for flexible tessellation and interlocking of adjacent columns, forming intricate, quasi-continuous wall surfaces using a single repeating component. A generative shape grammar is developed for parametric wall surface description and validated against desktop-scale prototypes. A building-scale prototype is also constructed from cardboard material, showcasing the use of integrated digital design and fabrication techniques for the rapid construction of ultra-lightweight curved-crease origami building systems.

Keywords: Curved-crease origami; Modular construction; Origami-inspired engineering

Designing Curved Folded Structures through Topology Optimisation

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Curved folded structures, created through folding, bending, and joining non-rigid sheets, are widely employed as decorative components. However, their functional applications remain limited, as the modelling process can be time-consuming and computationally intensive, and designing structurally sound curved folded structures presents another layer of complexity. This study introduces a new type of curved folded structure, which can be automatically designed by topology optimisation algorithms through stiffness maximisation and straightforwardly constructed using simple curved folded sheets. The design technique uses the mirror reflection principle to transform optimised topologies into their corresponding curved folded forms for efficient load paths. A series of examples are presented to demonstrate the effectiveness of the design-to-reality workflow. Finite element simulations are performed to reveal the performance of optimised curved folded structures.

Keywords: Topology optimisation; Curved-crease origami; Structural design

Actuating tubes in multilayer curved folding

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This paper presents possible ways to apply and integrate foldable tubes in combination with curved foldlines. The authors briefly present and explain the idea of tubes in relation to surfaces and discuss different approaches. The range of topics start from tubes that help actuate and form curved surfaces in a controlled way, over to pairwise tubes that can help to block mechanisms as soon as their relation is defined, to tubes that allow for a higher stiffness of folded configurations.

Keywords: Structural origami; Technical folding; Curved folding; Actuation; Applied folding

Slit-Folding — Actuating Curved Creases by Closing Tailored Openings

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In this paper, the authors will present a new strategy combining folding and kirigami approaches, entitled slit-folding. Cutting slits and removing (or adding) material along foldlines leads to interesting results and new folding mechanisms. The study of the possible slits' geometries and their relation to the adjacent foldlines allow for better control of these mechanisms that have a 3D minimal energy equilibrium state after stitching their seamlines. This paper will give an overview of this particular system.

Keywords: Structural origami; Technical folding; Curved folding; Kirigami; Fabrication; Slitfolding

Rigid-ruling Curved Folding Origami Implemented with Straight Inflated Air Pouches

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Rigid-ruling curved folding is a curved folding origami where each surface ruling remains rigid. This origami type exhibits a one-degree-of-freedom (one-DOF) mechanism beneficial for deployable structures. However, implementing mechanisms with panels requires many hinges and increases product weight. We implement rigidruling curved origami by inflating a planar pattern of slender air pouches along the ruling direction. Our approach has the advantage of using lightweight materials (air) with a simplified manufacturing process that can be patterned in a flat state. We also explore the feasibility of using the buckling behavior of the structures for self-folding of the mechanism.

Keywords: Curved folding origami; Air pouch; Pneumatic structure; Rigid-ruling; Mechanism

Topologically Variable and Volumetric Morphing of 3D Modular Origami Structures

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The morphing of 3D structures is suitable for i) future tunable material design for customizing material properties and ii) advanced manufacturing tools for fabricating 3D structures on a 2D plane. However, there is no inverse design method for topologically variable and volumetric morphing or morphing with shape locking, which limits practical engineering applications. In this study, we construct a general inverse design method for 3D architected materials for topologically variable and volumetric morphing, whose shapes are lockable in the morphed states, which can contribute to future tunable materials, design, and advanced manufacturing. Volumetric mapping of bistable unit cells onto any 3D morphing target geometry with kinematic and kinetic modifications can produce flat-foldable and volumetric morphing structures with shape-locking. This study presents a generalized inverse design method for 3D metamaterial morphing that can be used for structural applications with shape locking. Topologically variable morphing enables the manufacture of volumetric structures on a 2D plane, saving tremendous energy and materials compared with conventional 3D printing. Volumetric morphing can significantly expand the design space with tunable physical properties without limiting the selection of base materials.

Keywords: 3D modular origami; Topologically variable morphing; Volumetric morphing; Shape lockable morphing; Inverse design; Material design; Instability

Optimisation-based analysis of elastic buckling in cylinders with pre-embedded origami patterns

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Pre-embedded origami patterns in thin-walled cylinders have been shown to effectively guide the elastic deformation process towards predictable buckling mode with specific shapes. However, simulation tools for analysing such an interesting phenomenon remain underdeveloped, impeding a comprehensive understanding and its subsequent advancement. This study introduces an optimisation method for efficiently capturing the buckling behaviour under given boundary conditions through minimising elastic strain energy. The method is developed by representing origami surfaces as simple spring grids, where stretching and bending are captured by local elongation and rotation deformations. A series of pre-embedded origami patterns are analysed. The accuracy of the new simulation tool is first verified with experimental data, focusing on tubes buckled to their critical states. The tool is then used to reveal detailed intermediate behaviours throughout the buckling process.

Keywords: Thin-walled cylinder; Buckling shape control; Pre-embedded origami; Nonlinear programming

Finding the effective Poisson's ratios in stretching and bending of a reconfigurable morph origami

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This study investigates the Morph origami pattern, a recent generalization of Miura-ori and eggbox patterns. Each unit cell along a strip of the Morph pattern can exist in Miura-type or eggbox-type modes, giving rise to several hybrid configurations. It was previously shown that, when all cells are of the same mode, the in-plane stretching and out-of-plane bending Poisson's ratios of the Morph pattern are equal in magnitude and opposite in sign. Whether such an exotic relationship holds for the hybrid configurations of Morph is an open question. In this study, we propose a numerical approach based on bar and hinge model and homogenization to obtain the effective properties of the Morph pattern. Using this approach, we obtain results which indicate that the relationship between stretching and bending Poisson's ratios holds across the hybrid Morph configurations as well.

Keywords: Geometric mechanics; Origami metamaterials; Homogenization; Poisson's ratio

Cyclic Testing of Membrane Hinges for use in Origamiinspired Engineering Design

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Membrane materials can effectively be used as hinges in origami-inspired engineering designs. Because this application of membrane materials is recent, an understanding of how these materials perform under repeated cyclic loading in these conditions needs to be defined. This research reports cyclic testing of membrane hinges for origamiinspired engineering designs. When incorporating membrane hinges, the life of the membrane material should be considered to ensure that the device will withstand the anticipated number of cycles. Testing includes three polymers, a fiberglass, and three steels (including a stainless steel mesh) with 0.076 mm thickness. They are tested under fully reversed loading conditions with a constrained minimum radius. Because of the low number of cycles anticipated in most deployable origami systems, the membrane hinges were tested up to 10,000 cycles. The number of cycles until visual damage and the number of cycles until complete failure were observed and recorded. The polymers and fiberglass exceed 10,000 cycles and cycle data is provided for the metals. The work can be used to guide testing of membrane hinges for use in engineering applications or origami, including deployable space systems.

Keywords: Membrane hinges; Surrogate folds; Origami-inspired engineering design; Fatigue life

Motion analysis of Flexible Modular Origami: A Finite Particle Method Investigation

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Modular origami is a type of origami where multiple pieces of paper are folded into modules, and these modules are then interlocked with each other forming an assembly. When the flexibility of the modular origami is considered, the motion trajectory can be changed and the motion can even be locked. Many numerical approaches have been proposed to simulate the motion of origami. Here we focus on the finite particle method (FPM) and derive a formulation for modular origami models. Based on the FPM, we simulate motion of modular origami. Both 2D and 3D examples are given to verify the proposed formulation. This work paves the way for achieving the controllable motion paths of the flexible modular origami structures.

Keywords: Modular origami; Non-rigid origami; Finite particle method

Kirigami-inspired rectangular iso-area twist tessellations in architecture

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Origami and kirigami techniques unveil artistic and structural synergies that establish a platform for innovative solutions in architectural design. This research presents a fusion of origami and kirigami principles in the manipulation of Momotani's Wall tessellations, specifically through the deliberate incorporation of kirigami-inspired incisions. The strategic application of these cuts in hand-folded prototypes refines the folding process by reducing the resistance to folding. This newfound flexibility allows the deployment of the structure whilst constraining selected facets to the plane.

Keywords: Iso-area; Rectangular twist; Kirigami; Architecture; Design; Momotani brick wall

Growing kirigami with self-healing and reprogrammable mechanical properties

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Kirigami structures have proven instrumental in unlocking diverse properties from conventional materials. Despite the exploration of various geometric patterns and materials in kirigami design, it is crucial to acknowledge that a single kirigami pattern typically exhibits limited configurations and fixed mechanical properties. To address this limitation and advance the functionality and durability of kirigami materials, we present a novel approach - crafting kirigami with a new type of dynamic polymer that we call growing polymer. The growing polymer, synthesized through ring-opening metathesis polymerization (ROMP) of methyl cyclopent-3-ene-1-carboxylate, possesses the unique ability to dynamically alter its size and mechanical properties. Strategically assembling the growing polymers with passive polymers forms the foundation of the growing kirigami. During the growth process, the kirigami is immersed in a monomer solution, initiating osmotic swelling that absorbs monomers into the growing polymer. Subsequent polymerization and chain exchange reactions integrate new polymers into the original network, resulting in macroscopic growth. This growth process vividly showcases the transformation of the kirigami structure from a flat 2-dimensional (2D) configuration into a dynamic 3-dimensional (3D) structure. Moreover, the compressive mechanical properties of the 3D kirigami can be reprogrammed through the growth process. Remarkably, the growing process can be repeated multiple times, offering on-demand reprogramming of the kirigami structure and mechanical properties. Additionally, the growing polymer exhibits a self-healing capability, effortlessly repairing fractures by rejoining two broken pieces and undergoing self-healing for 2 hours, resulting in a self-healing kirigami. The synergy between the growing polymer and kirigami design introduces a transformative platform for creating metamaterials characterized by unprecedented reconfigurability and reprogrammable mechanical properties.

Keywords: Kirigami; Reprogrammability; Self-healing

Fabrication of DCRA using Kiri-origami Structure

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We proposed a nobel fabrication method using a Kiri-origami structure for a dihedral corner reflector array (DCRA), a floating image projection device. A Kiri-origami structure means a thin film with folding and cutting lines. We folded the Kiri-origami structure by stretching, and then compressed it to fabricate an orthogonal mirror array structure. We clarified that the aspect ratio of the mirrors is important for the proposed fabrication method, and we found that an aspect ratio of around 2.0 is desirable for the fabrication. Based on these results, we have successfully fabricated mirror arrays using the proposed fabrication method.

Keywords: Kiri-origami; DCRA; Honeycomb Core

A kirigami-inspired folding method for Miura thick panel

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This paper investigated the folding method of the Miura thick panel and obtains the basic cell of the Miura thick panel by introducing cuttings. Based on the Rodrigues spinor theory, the motion path of the basic cell was analyzed. The relationship between the coordinates of feature points and the folding angle was obtained, and it was proved that the basic cell of the Miura thick panel is a four-degree of freedom(DOF) unit. The connection rules on the two basic cells are studied and the folding feasibility of basic cells under different connection rules is discussed. To extend the topology of the basic cell, an 8R-Loop element was introduced to eliminate the redundant DOF of the 2*2Miura thick panel topology element, and the overall folding performance was analyzed.

Keywords: Kirigami; Miura origami; Folding; Thick panel; Configuration design

Rotational Erection System (RES) variations: fractals, tessellation and interlinkage

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Rotational Erection System (RES) is a design method to make three-dimensional (3D) structure from a single sheet with systematic cuts and folds. It is Origami extended with cuts or Kirigami folded into 3D shapes. We demonstrate an advanced applications using a number of RES units in fractals, tessellation and interlinkage. The design variations in simulated images and physical models shall exemplify the possibilities of both functional and aesthetic applications.

Keywords: Fractal; Kirigami; Linkage; Pop-up; Tessellation

Folding Condition of Kirigami and Rigid-foldable Kiriorigami Structure with Periodic Incisions on Concentric Circles

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A kirigami structure with periodic incisions on concentric circles is known to largely deform in the out-of-plane direction. By adding the specific folding lines to the concentric kirigami structure, we revealed that the concentric kirigami structures with the folding lines (referred to as a "kiri-origami structure") can be folded rigidly. Based on this rigid-foldable kiri-origami structure, we considered a theoretical model that generalizes the kinematics and obtained the relationship between the out-of-plane deformation and the radius change. Finally, we compared the theoretical model and experimental deformations by changing the cutting ratio of the folding lines.

Keywords: Kirigami; Kiri-origami; Rigid folding; Flexural rigidity; Out-of-plane deformation

Design of morphing and multifunctional shape profiles through cutting tessellations

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Miura-ori tessellations, known for their simplicity and single degree of freedom, can be ingeniously reshaped by strategically introducing cuts along and across the crease pattern edges to introduce extra degrees of freedom. This article explores the design and manufacturing aspects of cut-and-fold tessellated solids of revolution, emphasizing their ability to maintain reasonable stiffness and cushioning properties. The primary focus is on creating tessellations suitable for large-scale industrial production and packaging design applications. A family of Miura-ori kirigami derivatives are explored primarily through paper models with some supporting mechanical simulations to illustrate the compliance gained by introducing cuts to the underlying crease pattern. Our interdisciplinary approach integrates quantitative analysis through mechanical simulations, providing new understanding of the structural and functional characteristics of the proposed tessellations. By optimising the cut-and-fold process, we aim to enhance the adaptability of Miura-ori structures for practical use in industries requiring robust packaging solutions. This work contributes to the advancement of tessellation design methodologies and demonstrates the potential for transformative applications in the field of industrial packaging.

Keywords: Miura-Ori; Kirigami; Design; Packaging materials; Industrial production

Auxetic Kirigami Pattern inspired by Indusium in a Dictyophora Indusiate

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Dictyophora Indusiata has a network structure called indusium. This Kirigami-like structure is folded compactly into the pileus. Each cell of the structure deploys by rotating the Y-shaped components. However, the geometrical patterns of the indusium are not clarified. Here we show how to design deployable patterns from the network structure of indusium using Kirigami techniques. We anticipate applying geometrical patterns of indusium to achieve the efficient manufacturing of a deployable structure. For example, it is possible to manufacture huge and complex three-dimensional structures without large factories by cutting slits in flat sheets.

Keywords: Biomimetics; Auxetic; Kirigami; Algorithmic design; Deployable structure

Mechanical Properties of Gradient Miura-ori Metamaterials

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Miura-ori metamaterials are lightweight and porous structures and their mechanical properties are determined by their folding modes and geometries. In this study, the influences of thickness gradient mode (TG) and acute gradient mode on the energy absorption capacity of Miura-ori metamaterials are studied. Finite element analysis of the mechanical properties of the metamaterials is conducted. The results show that metamaterials with negative TG have better energy absorption capability, and the average force and energy absorption of metamaterials with negative TG are better than those with the acute gradient.

Keywords: Miura-ori metamaterial; Gradient mode; Thickness gradient; Finite element analysis

Parallel Session II: Engineering 2

A Flat Foldable Solid Consisting of Rhombitruncated Cuboctahedra and Regular Octagonal Prisms

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We propose a method for creating flat foldable solids by regularly removing panels from space-filling polyhedra. We classify space-filling polyhedra by characteristics. From the classification, we focus on a space-filling polyhedron consisting of rhombitruncated octahedra and regular octagonal prisms, which have similarities with truncated octahedra. Based on the case study with truncated octahedra, we show the solids and developments made from the two types of removing panels patterns and describe their characteristics.

Keywords: Flat foldable solids; Origami; Space-filling polyhedral

Kinematic Modeling of Cylindrical Origami Tessellations for Programmable Local Motion Control

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Origami has been intensively investigated due to its unique properties of changing designed planetary configurations to fit curved surfaces and compact forms. In this paper, the cylindrical origami structures based on generalized Miura-ori tessellations are studied, including geometric parameters, folding behaviors, and target curvature fitting. The design of the in-plane crease patterns to make the origami approximately fit a target cylindrical surface is first presented based on the out-of-plane method and parametric modeling. Multiple free parameters are undetermined while fulfilling the geometric constraints to realize this objective. Next, we systematically studied different folding behaviors and their resulting geometric patterns by varying these free parameters. Finally, methods are presented to locally modify the deployment kinematics while fitting the same target surface. This research extends the understanding of origami behaviors to full-period folding processes, which can be potentially applied in diverse engineering applications.

Keywords: Miura-ori; Kinematics modelling; Cylindrical surfaces; Local motion

Fahrenheit 1832: Folding for Fire Protection

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We present the research and development of fire-resistant sandwich structures using a mica-based inorganic paper core. The Miura-ori geometry of the core provides good strength and lightweight performance while allowing the relatively brittle fire-resistant mica paper to deform non-destructively. Combined with a geopolymer foam binder, the structure can withstand temperatures up to and beyond 1000°C for extended periods at a fraction of the weight of current protective systems.

Keywords: Origami; Tessellations; Sandwich; Lightweight; Fire-protection; Insulation

Topological manifold based parametric design of chiral origami mechanisms

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Chirality is an inherent property of the nature, referring to the asymmetric property of an object, which represents an effective means of achieving compactly arranged structures bestowed by biological evolution. It opens up a novel avenue for designing programmable helical structures capable of switching between rotational and translational movements by the exhibited mirror-symmetric characteristics. This paper focuses on the topological manifold based parametric design of chiral origami mechanisms, combining the excellent folding features and periodic tessellation of origami structures. The mechanisms exhibit mirror-symmetric characteristics, allowing the transition among two distinct fully-folded configurations and the unfolded configuration. The proposed parametric design theory for chiral origami mechanisms combines the characteristics of spherical manifolds without length scale constraints, conducting the angle synthesis with tuning 2D planar profiles and 3D motion space using only one design parameter. The proposed theory reveals the mathematical essence of chiral origami mechanism design, elucidates the intrinsic connection between key angle parameters and comprehensive mechanism design, and formulates a unified mathematical model for a class of chiral origami mechanisms units. It lays the foundation for the subsequent development of reconfigurable modular chiral origami robots with diverse motion capabilities and remarkable ability to adapt to various tasks and changing environments.

Keywords: Parametric Design; Chiral Origami Mechanisms; Topological Manifold; Angle Synthesis

From Flexagon to Flexahedron – Infinitely Turning Objects

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Flexagon is an interesting two-dimensional geometric object that can be turned inside out through three-dimensional rotations. It is usually obtained by folding a long strip of paper into identical shapes such as equilateral triangles, squares, heptagons, or hexagons, etc., and taping the two ends of the paper strip to form a closed loop. In this paper, a three-dimensional cuboid flexahedron is created based on its origami relative, tetraflexagon that consists of a loop of squares. The flexahedron is built by connecting identical cuboids (square prisms) through a special hinge-shifting mechanism. It demonstrates infinite turns whose faces can be hidden and revealed in a cyclic order through continuous flex moves. The kinematics of a tetraflexagon and a cuboid flexahedron are analysed and their cyclic motions are compared afterwards.

Keywords: Flexagon and flexahedron; Infinite turns; Cyclic motion; Kinematics; Hinge-switching mechanism; Reconfigurable origami

Theoretical Analysis on the Deformation of the Miura-Ori Patterned Sheet

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A theoretical analysis is conducted on the deformations of the Miura-ori patterned sheet. The sheet deforms within the elastic range of the material Elvaloy in the mechanical tests, which includes the out-of-plane compression, in-plane compression and three-point-bending tests. Among other assumptions, the Miura-ori's one degree-of-freedom mobility and rigid facets are kept in the theoretical model for the simplicity of the analysis. The global deformations of the patterned sheet are integrated by the elastically deformed portions along the ridgelines. The relationships between reaction forces and displacements are obtained using the principle of virtual work. In energy balance equations, external works are done by reaction forces and internal works are generated by elastic deformations along ridgelines. The theoretical results show good correlation with the experimental data, except in the in-plane X1 compression and three-point-bending X1. The disparities between the theoretical and experimental data indicate a refined model is needed for a better description of the corresponding deformations.

Keywords: Origami; Miura-ori Sheet; Theoretical Analysis\

Deformable Origami Structure Design Based on Two-Dimensional Geometric Face Shape Collocation

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The present study introduces an innovative origami structure design technique that utilizes precise two-dimensional face shape design and crease arrangement to create three-dimensional deformable structures with negative Poisson's ratio characteristics. The significance of symmetry in the design process is emphasized by our study, which also explores potential applications of these structures in engineering and design fields. The objective is to demonstrate the innovative potential that arises from the fusion of origami art and engineering.

Keywords: Origami; Deformable Structure; Geometric Shape; Metamaterial; Negative Poisson's Ratio

Parametric Study of the Porous Origami-based Mechanical Metamaterials with Curvatures

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Light-weight, high-strength metamaterials with excellent specific energy absorption (SEA) capabilities are significant for civil engineering aerospace engineering and automobile industry etc. However, most of the mechanical metamaterials are investigated and manufactured in the shape of polyhedrons, hence limiting the application of mechanical metamaterials. Here, inspired by the foam-like protective layer of fruit, the pomelo peel, the cylindrical and spherical origami-based porous mechanical metamaterials are studied, the numerical model is validated through the experiments, and the effects of the major parameters of the structures are investigated to understand the effects of the energy absorption capacity of the structures. The research here offers a reference for designs to be used in future applications such as sports helmets, protective layers and unmanned landers.

Keywords: Porous mechanical metamaterials; Specific energy absorption; Cylindrical; Spherical; Protective layer

Earwig Fan Inspired Deployable Structures

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Foldable wings in certain insects represent the ultimate deployable structure, capable of rapid folding/unfolding while possessing the strength and rigidity to withstand high-speed flapping. Among these insects, earwigs use a unique fan-like folding mechanism, allowing for the most compact wing folding in insects. The authors have elucidated a geometrical design method to apply the intricate folding patterns of earwig fans to various forms of deployable structures. In this paper, we show new deployable structures developed through biomimetics, including paper fans, tents, deployable roofs, and providing insights into deployable solar panel arrays designed for lunar base applications.

Keywords: Biomimetics; Deployable Structures; Algorithmic Design; Grasshopper

Origami Cellular Material Switching Between Single and Multiple DOF Modes

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Cellular structures made by assembling origami attract attention because of their functions, such as auxetic behavior, anisotropic stiffness, and flexibility. This study proposes an origami cellular material inspired by BuildVoid that can switch modes between single and multiple degrees of freedom (DOF). The mechanism consists of modules with square faces repeated in two directions. By switching the folding mode of each module, the synchrony of the modules switches, which allows the switching of DOF of the whole mechanism. This paper clarifies the kinematics of the mechanism. We believe that the proposed materials with stiffness-switchable properties can be applied to robotics, adaptive materials, and programmable matter in the future.

Keywords: Origami metamaterials; Switching degree of freedom; Rigid origami

A flat-foldable, transformable metamaterial from octahedral origami unit cells

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Foldable metamaterials are often used to create deployable structures with unique mechanical properties, such as programmable stiffness, reconfigurable topology, and large volumetric change. These metamaterials are constructed from individually transformable unit cells that are interconnected to form shape-changing macrostructures. In this work, we explore an origami-inspired stellated octahedron unit cell with three-dimensional flattenability and propose a kinematic characterization method based on the unique geometry of a unit sphere passing through a surface plane. Based on this unit cell, we propose a new class of reconfigurable 3D lattice metamaterials that can fold flat across multiple dimensions. We establish the energy profile of a single unit as a function of spring forces acting on the design, providing a framework for engineers to tailor the energetic behavior of the system on both an individual and macroscopic scale. From these base kinematics, we introduce a set of connective topologies to create metamaterials with varying shape-changing properties in a three-dimensional lattice. We also introduce a method for generating valid 3D tessellations and enforcing constraints of local and global connectivity across modules.

Keywords: Transformable; Metamaterial; Octahedral; Lattice; Three-dimensional

Kresling-Inspired Constant Size Magnetically-Reconfigurable Metamaterials

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¹ Princeton University, United States

The field of origami engineering has explored many multistable structures. One such structure is the Kresling origami, which is a pleated cylinder that can have stable deployed and compressed states. Much research has been done to understand the kinematics and mechanics of the Kresling unit cell. In comparison, the mechanics of towers consisting of stacked Kresling cells has been studied less. Here we present a simple model which can accurately predict the load-displacement behavior of Kresling towers consisting of multiple, potentially distinct Kresling unit cells. In addition, the model reveals the presence of multiple stable states for certain tower heights. We then apply this theoretical understanding of Kresling towers to design a magnetically-reconfigurable metamaterial which exhibits tunable stiffness while maintaining a constant form factor.

Keywords: Metamaterials; Tunability; Magnetic Actuation

Design and Fabrication of Quasi-isotropic Origami Metamaterials

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The objective of this study is to design a metamaterial with quasi-isotropic mechanical properties, capable of being mass-produced. We propose a metamaterial in which a unique tetrakaidecahedron is tessellated in space. The metamaterial can be fabricated by 3D printing as numerous researchers have done Kelvin cell metamaterial; however, we focused on an origami-like method by folding a flat sheet and an industrial manufacturing method by pressing and stretching a flat sheet. As these methods can produce metamaterials rapidly and continuously compared to 3D printing or stereolithography, the metamaterials could be implemented easily as a part of mechanical devices. Finite element analysis on the proposed metamaterials revealed that the amount of absorbed energy under dynamic compression in the out-of-plane direction was approximately equivalent to that in the in-plane direction as required. This finding indicates that the proposed metamaterials can solve a drawback that stiffness of conventional core materials (e.g., honeycomb cores) is not isotropic and energy absorption capability is highly affected by impact directions.

Keywords: Composite material; Structure design; Energy absorption; Crush worthiness

Reconfigurable Mechanical Logic Metamaterial

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Most of the mechanical logic metamaterials/structures are bistable/multi-stable, and few existing works have been found to realize mechanical logic from the mechanism perspective. Here, we design a conductive module metamaterial inspired by a planar 4R mechanism to achieve mechanical logic based on its reconfigurability. By utilizing the motion bifurcation to realize the transformation between configurations, we realize logic gates and corresponding combinations (including half/full binary adder/subtractor). Since elastic deformation occurs only on hinges, the module can automatically return to its initial state after one computation for reuse. This work provides a new idea for the design of mechanical logic/computation.

Keywords: Reconfigurability; Mechanical logic; Planar mechanisms

Adding more bite to the origami chomper

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We begin with Jeremy Shafer's Chomper, a playful kinematic origami gadget with a two degree-four vertices. Pressing both actuating points of the symmetrical gadget causes a set of jaws to chomp, or open and close. We then explore the oriceps, an origami forceps design derived from the chomper. From an origami design perspective the oriceps is a chomper gadget with a strip graft that splits the vertices along the central mirror-axis crease. The resulting gadget has four degree-four vertices with two additional affordances, a flat gripping point and a flat area for attaching a linkage mechanism facilitating the manipulation of the jaws. The chomper and oriceps gadgets actuate two 'jaws' and at this point we ask, "can we add more 'jaws', or bite, to this gadget?" The answer is yes. A chomper with more jaws is indeed possible! We show that adding more jaws is possible by rotating the jaw mechanism design around the central axis. Enter the three-jawed Thysan gadget, named after the distinctive triangular petals of the Australian native Thysanotus genus within the family Asparagaceae. This gadget features six degree-four vertices. We present a geometric study of the Thysan gadget through a parametric design model. The model facilitates interactive design and visualisation and implements recent advancements in rigid origami for degree-four vertices and rigid origami kinematic equations.

Keywords: Origami gadgets; Origami robotics; Origami chomper; Jeremy shafer; Oriceps; Rigid origami; Rigid origami kinematics

Research on construction of double cubic core and its application

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A cubic core is a structure made by folding a single plate with square holes at equal intervals punched on it. Like a honeycomb core, it is lightweight, thick, and has an excellent structure with a certain level of strength. We first propose a method of creating a cubic core by assembling (or weaving) paper strips into a grid pattern instead of punching holes. This method involves preparing paper strips of a certain width, arranging them at equal intervals, gluing them, and folding them. The advantage of this method is that there is less waste compared to normal methods, and parts are easier to manufacture. One of the characteristics of a cubic core is that its top and bottom surfaces form an uneven checkerboard pattern. It can be better to have a uniform height on all surfaces. If the checkerboard patterns of two cubic cores can be stacked one on top of the other, by filling in the recesses with each other, it is possible to create a cubic core with the same height on all sides. We introduce it as a double cubic core. A double cubic core cannot be made by combining two plates with holes. We propose some methods to create the double cubic core. Depending on the method, it is possible to create arbitrary dot patterns or restricted dot patterns. We show a design of a font using such a constrained dot pattern as an application of the double cubic core.

Keywords: Cubic core; Double cubic core; Feasibility of construction; Font

Kinematics Analysis of Rubik's Magic Puzzle and beyond

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The Rubik's Magic puzzle is a mechanical toy comprised of eight thin panels. Connected through a unique cord-based mechanism, the panels can fold along two perpendicular directions, achieving various reconfigurations by switching its rotational axes. This research unravels the hinge-switching process by analysing the puzzle's stringing patterns and investigating the kinematics of the panel-cord assembly on two thin panels – it can be described by the bifurcation of a spherical 4R linkage. A physical model demonstrates continuous rotation and flipping motion between the pair of thin panels. Furthermore, the study extends its scope by varying panel geometry and thickness on the reconfiguration process. The findings serve as a groundwork for future research in designing cord-based mechanisms to achieve hinge-switching properties for engineering applications.

Keywords: Rubik's Magic puzzle; Kinematics analysis; Cord-based mechanism; Hinge-switching mechanism; Reconfigurable origami; Modular origami

New Kresling Origami Geometry: The Offset Cell

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The Kresling cell is an origami-inspired structure that can undergo a large, bistable shape change. We modify the standard Kresling cell such that its top and bottom faces are non-concentric in the folded state, and angled in the expanded state. This new geometry maintains the bistabillity and geometric relations of the original cell while unlocking new kinematics and ways of tuning cell energy barriers.

Keywords: Origami; Kresling Origami; Bistabillity

Making Origami Digital Musical Instruments

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This paper presents an experimental study in post-digital luthiery using origami geometries. We explore how our method of fold sensing creates a new class of digital musical instruments through a case study of music and visual composition and performance. We consider how single-fold and multi-fold gestures afford an exploration of sound as a space, where each fold-sensed crease creates a new variable dimension in the musical parameter space, and multi-fold gestures allow multiple parameters to be controlled through origami gestures. We discuss our fabrication method, including material selection, sensor construction, and the selection of origami patterns. The raw sensor data from the instrument are interpreted with our fold sensing method that can infer four distinct but related types of gestural interaction, including touch, pressure, single-fold sensing, and multi-fold sensing. The data stream conforms to digital music standards, MIDI (Musical Instrument Digital Interface) and OSC (Open Sound Control). We introduce our open-source software toolkit, designed for fostering artistic experimentation, that implements and supports the musical standards. Our case study includes three distinct artistic strategies for interpreting the data stream and discusses the types of gestures employed by each artist in composition and live performance. We conclude that origami gestures show distinctly new types of expressions for both music and visual digital artists.

Keywords: Origami music; Music; Fold sensing; Oribotics; Midi; Open sound control; Multi-fold gestures; Single-fold gestures

Geometric Constructions of Bifoldable Polyhedral Complexes

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We introduce a general geometric framework for the construction of polyhedra and polyhedral complexes that are bifoldable, i.e. bi-directional, flat-foldable into two orthogonal planes. This vastly generalizes origami folds known as the Miura pattern, the origami tube and the Eggbox pattern. Our polyhedra are generalized zonohedra based on 1-parameter family stars of vectors in R³ that deform in specific ways while the polyhedra and polyhedral complexes are flat-folded. After describing the framework, its basic features, and the general design process, we give several new examples of infinite doubly periodic, triply periodic, and fractal bifoldable polyhedra.

Keywords: Bifoldable; Zonohedron; Polyhedron; Origami; Miura

Fold Sensing Origami Gestures - A case study with Kresling Kinematics

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We extend previous work in the field of origami robotics by exploring the question of Fold Sensing. Fold Sensing is the process of sensing the angle between two adjacent folding planes and can be used to approximate an origami structure as it folds and unfolds. Our work shows that capacitive sensing can be used to detect four distinct types of interactions from the same sensor data stream. Our capacitive Fold Sensing design can detect touch as a boolean state, pressure as a scalar value, single-fold gestures as a scalar value measured between two panels, and multi-fold gestures as a multi-dimensional vector, comprised of single-fold values. Our case study considers single and multi-fold gestures for the unique origami kinematics of the Kresling-ori. We present a prototype fold sensing instrument and two digital twins. The unique folding kinematics of the Kresling-ori has distinct folding gestures such as compression, rotation, twisting, and bending, making it ideal for this case study. We detail our methods for translating sensor data into gestural data for each type. We include a partial characterization of the sensor design, sensor calibration process, single-fold and multifold gesture recognition methods. Our study points to further work towards a generalized model for sensing the complex kinematic structures in origami, in rigid and flexible origami structures.

Keywords: Fold Sensing; Capacitive Sensing; Conductive Textiles; Origami; Origami-Gestures; Kresling-ori; Kresling; Multi-fold Gestures

Design and development of a foldable and transformable hemispherical enclosure for robotic manufacturing

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Advanced digital manufacturing technologies are currently under development in laboratories or specific fabrication environments. In the construction sector, numerous robotized processes are being experimented with, but their on-site implementation remains challenging. A potential solution is to decentralize production through modular laboratories or portable enclosures that can be brought to the execution site, enabling resource-savings and efficiency. Transformable enclosures allow flexibility in usage by providing shielding and security as well as accommodating design demands such as weatherproofing and noise, visual and splinter protection when needed. This project presents a concept of a hemispherical origami-based transformable and lightweight hybrid enclosure that can be applicable to robotic fabrication applications. Automated design processes implemented origami-based patterning techniques, identifying and adapting the origami pattern to gain full transformability. Static and dynamic analyses, as well as Multi-Body simulations, were carried out, to verify foldability and stability. The outcame is a realized construction of 2.3m height, 4.15m diameter that houses a robot with a 1.65m range and a 15kg payload demonstrating the feasibility of an origami-based enclosure system for architectural scale.

Keywords: Rigid origami; Hybrid transformable construction; Robotic fabrication enclosure; Design exploration; Finite elemente

Chiral origami robot with wheeled and quadcopter modes

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Wheeled robots have broad applications in even and ruled terrains due to their stable motion, simple control, and the high speed they can achieve. However, wheeled robots encounter many difficulties in rough terrains with large obstacles, which require complex route planning to adapt to these scenarios. This paper introduces a variable-mode robot inspired by chiral origami, which can switch between wheeled and quadcopter modes. The origami-inspired structure has one degree of freedom, which allows it to perform folding motion for mode transition using a single actuator. The structure inspired by animals that roll into balls is added to the robot, which can serve as a landing buffer for the quadcopter and form a wheel structure in the rolling mode through the folding motion. The robot in this paper can adapt to various scenarios using different modes through folding and can effectively resolve issues related to traversing rough terrains and navigating large obstacles.

Keywords: Origami robot; Chiral origami; Variable-mode; Biomimetic

A robotic origami folder for paper cranes

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This paper presents a robot system to fold paper cranes automatically. It consists of a cutting plotter, two robot arms, and a base for papers. An origami paper with creases is prepared by the cutting plotter. Then the paper is placed on the base and folded by the arms. Suction pads on the base are selectively activated to fix the paper in the folding process. Paper folding is achieved with specially-designed tools for the robot arms without sensor feedback. Now the system can fold complete paper cranes, which include squash, petal and inside reverse folds.

Keywords: Robot; Automation; Paper crane

Miura-Bot: Modular Origami Robots for Self-Folding Miura-Ori Tessellations

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We present Miura-Bot, an open-source modular origami robot design that leverages actuated multimodal origami structures for diverse task-oriented scenarios. Miura-bot uses the Miura-Ori tessellation pattern for self-contained actuated locomotion and folding through coupled rigid panels. The design includes a 3D-printable parallelogram-shaped mechanical chassis and electronics modules for onboard computing, sensing, and actuating. Our experimental results demonstrate the locomotion of individual parallelogram robots and self-folding capability of one assembled unit of the Miura-Ori pattern made of four parallelogram robots. Future work will focus on algorithmic self-assembly, self-coupling capability, and experimental results on the form and function of various assembly patterns.

Keywords: Robotics; Modular origami; Modular robotics; Miura-ori pattern; Actuated origami; Origami engineering

Adaptive Stiffness and Shape Control of a Modular Origami-Inspired Robot

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This paper presents a combined origami-tensegrity modular system for the design of flat-foldable robots. Pre-tensioned cables integrated in the origami-inspired structure control both the deployed shape and the structural stiffness. The variable shape and stiffness allow for adaptation to uncertain environments, which can be challenging for conventional 'stiff' robots. The modular design system is based on Miura-ori tubes and allows for robot designs tailored to specific missions. A particular example robot concept is presented, which demonstrates shape authority and stiffness modulation. We describe an analytical model for structural stiffness for such robot concepts.

Keywords: Tensegrity; Adaptive stiffness; Modular; Robotic; Miura-Ori

Re-programmable Matter by Folding: Magnetically Controlled Origami that Self-Folds, Self-Unfolds, and Self-Reconfigures On-Demand

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We present a reprogrammable matter system that changes shape in a controllable manner in real-time and on-demand. The system uses origami-inspired fabrication for self-assembly and repeated self-reconfiguration. By writing a magnetic program onto a thin laminate and applying an external magnetic field, we control the sheet to self-fold. The magnetic program can be written at millimeter resolution over hundreds of programming cycles and folding steps. We demonstrate how the same sheet can fold and unfold into multiple shapes using a fully automated program-and-fold process. Finally, we demonstrate how electronic components can be incorporated to produce functional structures such as a foldable display. The system has advantages over existing programmable matter systems in its versatility and ability to support potentially any folding sequence.

Keywords: Origami; Self-assembly; Magnetic actuation; Reconfigurable robotics

Tessellation Manufacture by Sequential Quasi-Isometric Gradual Folding

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To facilitate the large-scale creation of origami tessellations for technical applications, a new manufacturing approach is proposed where the material is formed gradually. One pair of moulds is used for the stepwise folding process which introduces a constant increase in deployment degree – starting from the flat sheet material at one end towards a fully deployed state at the other end. The resulting stepwise forming process does not conform to a strictly rigid folding, but tries to locally minimise elastic deformation to enable globally large, quasi-isometric shape changes in the folded sheet. To find such a geometry for a gradually folded Miura-Ori tessellation, a numerical model was created using a dynamic relaxation approach with given boundary conditions for isometry and folding angles. Real-world folding experiments using corresponding additively manufactured moulds were conducted with different materials including papers, plastic and metal foils and the quality of the results were analysed.

Keywords: Manufacturing; Miura-Ori Tessellations; Gradual Folding; Quasi-Isometric Folding

Programming Origami Instabilities via Topology Optimization

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Instabilities have been commonly prevented in traditional engineering design. However, recently, origami-based instability mechanisms have been explored to design novel material or structural systems, e.g., Miura-ori-inspired morphing metamaterials, square-twist-based programmable metasheets, and Kresling origami-inspired highdamping devices. To overcome limitations of the origami design space, we seek to use the ground structure approach to explore potential crease layouts for an origamiinspired design with instabilities. The goal is to inverse-design optimal crease skeletons (truss structures) with programmable snapping instabilities. From an algorithmic viewpoint, we use a min-max topology optimization formulation subject to prescribed nonlinear structure responses (i.e., snapping equilibrium path). The objective consists of minimizing the errors between actual and prescribed load factors under given deformation. To capture the snapping equilibrium path in structural analysis, we adopt a modified generalized displacement control. We verify that the method can capture the equilibrium paths with prescribed snapping instabilities (snap-through and snap-back behaviors). Several two- and three-dimensional examples demonstrate the capabilities of the proposed formulation for programming structural instabilities. The present rodbased optimized designs can be reconfigured, which leads to desired functionalities, such as programmable snapping sequences and tunable mechanical responses.

Keywords: Non-rigid Origami; Instability; Topology Optimization

Cut Design of Pop-up Origami with Fixed Planar Substrate

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Pop-up designs serve as an efficient method for transforming 2D planar sheets into 3D structures. When the substrate maintains its original planar configuration, the design gains the ability to independently extend across the surface. Here, the Rotation Erection System (RES) is employed as the foundational concept to attain localized pop-up behavior. Stretchability analysis is conducted to establish the feasible connection boundary for the predefined rotation erection region. All creases are designed based on the predefined maximum rotation angle, validated through paper models with different geometry designs. Furthermore, the bistability behavior is undertaken from the perspective of incompatibility. This investigation extends to the incorporation of additional creases in the design, aiming to achieve multiple stable states.

Keywords: Pop-up origami; Cut design; Crease design; Analytical model

3D tissues using truncated octahedron blocks produced by Origami and micro/nano processing technologies

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Three-dimensional (3D) cell culture is widely employed for fundamental research in cell biology, drug discovery, tissue engineering, and regenerative medicine. However, methods for constructing 3D microstructures with cells are too complex and slow for mass production. Here, we have developed a technique to easily create 3D cell microstructures by exploiting the cell traction force (CTF) to drive the self-folding of microfabricated flat plates. This technique, called "cell origami", does not required external driving forces, such as thermal actuation or electromagnetic forces, which are required when constructing conventional 3D microstructures. We use the CTF to fold hinges made from microplates jointed by adherent cells. Various 3D cell-laden microstructures were successfully produced by controlling the shapes of the patterned 2D plates and the folding angles. We assemble microstructures like blocks to construct larger structures. We consider the optimal block shape that will prevent blocks from shifting or collapsing during assembly. When assembling blocks efficiently, it is better if the blocks can fill a space, and a truncated octahedron is a well-known block that can fill a space. We use truncated octahedron blocks to make larger 3D tissues. It is considered possible to achieve high throughput device that can be applied to future new drug development. Mass production technology for the substrates is progressing, and we do hope you will use it in your research.

Keywords: Cell origami; Truncated octahedron; Self-folding; Self-assembly; 3D shapes

Integrated non-destructive development design of oblique cutting four pyramid frustum origami shading cover

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The utilization of origami structure in the design of large-scale shading cover is prevalent, owing to its attributes of adaptable configuration, light weight, and elevated folding efficiency. An integrated crease scheme was devised for a 7150 mm tall, oblique cutting, four-prism-shaped film shading cover. Subsequently, while contemplating the structural boundary conditions and the cumulative impact constraints of thickness, MATLAB R2023 was employed to ascertain the comparative and selective design of the film shading cover's structural configuration. This process culminated in the determination of an optimal crease design, maximizing storage efficiency. Ultimately, the overarching deployment process of the large-scale thin film structure was analyzed and optimized, incorporating considerations of miniaturization, energy transformation, planarity, driving force, and other pertinent indicators.

Keywords: Large-scale shading cover; Configuration design; Creases optimization; Deployment analysis

Symmetric self-folding of N-gon hypar origami

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The hyper origami is folded from concentric pleated polygons, whose folded configuration is characterized by a negative Gaussian curvature. Experiments show that the hyper origami with N-sides exhibit multiple symmetric stable states. When N=4, the folded square hyper pattern is proven to converge to a standard hyperbolic paraboloid shape, with two symmetric states. When N=6, the hexagonal hyper pattern demonstrates two distinct configurations that follows different symmetry groups. In this research, we systematically look into the bifurcation configurations of N-gon hyper origami by means of self-folding. The simulation is performed based on a modified version of the MERLIN software that models nonlinear deformation of origami structures. We discover that the folded shapes of N-gon hyper origami is strongly related to the symmetry type of the initial perturbation. However, some configurations become energetically unstable as the amount of folding increases, and soon degenerate to stable configurations that break the symmetry of the crease pattern. We also conduct demonstrative experiments using smart material based laminates. This work paves the way to develop a unified mechanics model for N-gon hypar origami, and shed light on the possible applications of the hyper origami as shape morphing metasurfaces.

Keywords: Origami engineering; Hypar origami; bifurcation; self-folding; MERLIN simulation

Parallel Session III: Mathematics, Computation & History

Folding π

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It is well known that the set of origami constructible numbers is larger than the classical straight-edge and compass constructible numbers. However, the Huzita-Justin-Hatori origami constructible numbers remain algebraic so that the transcendental number π can only be approximated using a finite number of straight line folds. Using these methods we give a convergent sequence for folding π as well as other methods to approximate π . Folding along curved creases, however, allows for the construction of transcendental numbers. We here give a method to construct π exactly by folding along a parabola, and we discuss generalizations for folding other transcendental numbers such as $\Gamma(1/4)$.

Keywords: Origami; Constructions; Pi; Algorithms

Analysis of Huffman's Hexagonal Column with Cusps

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We analyze the mathematical existence of one of David Huffman's most prominent curved-crease designs: the Hexagonal Column with Cusps, featuring circular, parabolic, and straight creases. Observations of the physical folded shape suggest that the concave regions between two parabolas form a cylinder, and the regions between the circle and the nearest intersection of the parabolas form a cone. In our analysis, we deduce the remaining rulings that result in a numerically closed hexagonal shape. Finally, we explore other variations of the shape, including those that incorporate only circular creases.

Keywords: Folding; Curved-crease origami; Differential Geometry; Huffman's Hexagonal Column

A Characterization of the Overlap-free Polyhedra

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For a polyhedron Q, a polygon obtained by cutting the edges or faces of Q is called a general unfolding. A general unfolding may have a self-overlap or self-intersection on the boundary which depends on the way of unfolding. It is established by [Aronov and O'Rourke] and [Sharir and Schorr] that any convex polyhedron satisfies the property that at least one general unfolding has no overlap. This research focuses on a dual property in which any general unfolding has no overlap, called overlap-free. We show that a polyhedron is overlap-free if and only if it is a stamper, which is a notion introduced by Akiyama. This means that if a polyhedron is not a stamper, at least one general unfolding has an overlap. We prove it in a constructive way.

Keywords: Computational origami; Unfolding; Stamper

Continuous flattening of quadrangular prisms with all edges rigid except one

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There are many ways to flatten polyhedra continuously. This paper focuses on the maximum number of rigid edges in continuous flattening motions for a given polyhedron. First, we provide an upper bound of the number for each convex polyhedron based on the results described in the literature. Next, for some triangular prisms, we provide two flattening methods that keep more edges rigid. Moreover, using the two flattening methods, we provide continuous flattening motions for some convex quadrangular prisms with 11 edges rigid, i.e., all edges are rigid except for one. As a result, our method achieves the maximum number (precisely 11) of rigid edges in the continuous flattening of the convex quadrangular prisms.

Keywords: Continuous flattening; Rigid edge; Quadrangular prism

Continuous Flattening of the Surfaces of Regular Star-Polyhedra

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There are four regular star-polyhedra, known as the Kepler-Poinsot polyhedra, which are denoted by {5/2,5}, {5/2,3}, {5,5/2}, and {3,5/2} using the Schlafli symbol. Whether the surface of a polyhedron made of a flexible material such as paper can be flattened without cutting or stretching is a problem that has been investigated. This problem has been solved for all convex polyhedra by using moving creases to change the shapes of some faces, which follows from Cauchy's rigidity theorem. For non-covex polyhedra, the problem has been solved only for special polyhedra, such as orthogonal polyhedra, in general condition, that is, under the condition of finite creases in each folded state. The surfaces of regular star-polyhedra are not convex and they have not been included by known results, to the best of our knowledge. Thus, we give continuous flattening motions of the surfaces for all four regular star-polyhedra.

Keywords: Regular star-polyhedra; Flattening; Moving creases; Continuous folding; Non-convex polyhedral

On Beloch's curve that appears when solving real cubics with origami

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The Justin–Huzita–Hatori Axiom 6 of origami related to so-called neusis constructions assures the solution of real cubic equations Beloch showed in 1936. We investigate a certain real cubic curve F(x, y) = 0, say, Beloch's curve that appears in the algorithm and prove that its shape is determined by the sign of the Hessian HF = -4(4p+q 2) at its uniquely existing singular point P(p,q). This viewpoint would shed new light on the relationship between Axioms 5 and 6.

Keywords: Origami; Real cubic equation; Axioms 5 and 6

22.5-degree Reference Optimization via Continued Fractions

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The problem of reference finding in 22.5-degree crease patterns has received relatively little academic attention despite the growing interest in such designs. One procedure, developed by Fumiaki Kawahata and Hideo Komatsu, relies on similar geometry. The process is powerful and intriguing, but requires some trial and error, as the number of creases required depends strongly on the reference point chosen. In this paper, the author explores the use of continued fractions to determine which reference points in a crease pattern are best suited to Kawahata and Komatsu's technique. Along the way, the author builds on the work of [Tachi and Demaine 16] to characterize the vertices of 22.5-degree crease patterns not developed on a 22.5-degree grid.

Keywords: 22.5-degree; Origami; Reference finding; Continued fraction; Crossing diagonals

A Survey of Rep-Cube

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A polyomino is called a rep-cube if it can be folded into a cube and can be partitioned into two or more polyominoes such that each of them can be folded into a cube. This new notion was proposed in 2016 based on two famous ideas of polyomino and reptile. Since then, the author and colleagues have investigated several properties of this notion from the viewpoint of the folding problem. In this survey, we give known results in the previous work with some new ones.

Keywords: Polyomino; Rep-cube; Rep-tile

Star unfolding, source unfolding, and farthest point mapping on the 4-cube

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We draw a star and source unfolding of the 4-cube, and describe its farthest point mapping. The limit set of the farthest point mapping is the union of the diagonal line segments of the eight 3-cube facets.

Keywords: Source unfolding; Star unfolding; Farthest point mapping; 4-cube; Omegalimit set

Preliminary Research on the Bending Behaviour of Curved Origami in Shape Determination

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Curved origami has potential to be applied in the design of architectural geometry owing to unique configuration and the mechanical behaviour. This paper is focused on the mechanical behaviour in the shape design. If the fully developed flat situation is defined as the original state, the state in three-dimensional configuration is formed with bending deformation. Because curved origami is formed by developable surfaces, the mechanical properties including the bending deformation can be described by the principal curvature of the surfaces. Hence, the shape determination approach in this paper is especially concentrated on the surface curvatures.

Keywords: Architectural geometry; Curved origami; Bending deformation; Deployable surfaces

Finding Solutions to Origami Boundary Value Problems with Edges of Platonic Solids

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There are many interesting aspects to the boundary value problem of origami reconstruction, i.e. finding origami models folded from squares of paper in such a way that the edges of the folding square come to result in a given polyline. Such polylines must fulfill certain conditions in order for such a reconstruction to be possible. An interesting restriction is to consider polylines that are composed of edges of Platonic solids. In this talk, some results in this area will be presented, as well as some potential paths to solutions that have not (yet?) proven successful.

Keywords: Boundary value problem; Reconstruction; Platonic solids

Asymptotically-isometric deformations of periodic piecewisesmooth surfaces

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Origami tessellations and other periodic piecewise smooth surfaces admit almostinextensional deformations that are neither infinitesimally isometric nor finitely so (e.g., twisting motion of the Miura ori). Here, we propose and investigate a middle-ground: the concept of locally infinitesimal but globally finite inextensional deformations. These deformations are defined as asymptotically isometric to first order in the size of the creases rather than in the size of the displacements. The main result is a "coarsegrained Theorema Egregium" for periodic surfaces, namely a constraint on the effective curvatures of the periodic surface to be compatible, in a sense, with its effective metric. The theorem presumes periodicity, piecewise smoothness as well as an ansatz typical of two-scale asymptotics. At the heart of the proof is a self-adjointness property of the (linearized) Monge-Ampère operator. Various examples of curved-crease origami tessellations and periodic smooth surfaces are presented to illustrate the result.

Keywords: Curved-crease origami; Piecewise smooth surfaces; Inextensional deformations; Isometric deformations; Bending; Asymptotic analysis

Some applications of topology on origami

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In 2004, Demaine et al. showed that if P is a polygonal region in the plane, then for any folded state (f,λ) of P that is well-behaved of order 2, there exists a folding motion from P to (f,λ) . In this paper, we show that when P is homeomorphic to an annulus, there exist some folded states which admit no folding motion by using topological techniques, and give the necessary and sufficient condition to existence of folding motion from P to (f,λ) for P which is an annulus in R2 each of the boundary component of which bounds a polygonal convex region.

Keywords: Folded state; Folding motion; Topology; Knot and link theory

Computing Flat-Folded States

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In this paper, we introduce a facewise definition for global flat foldability on crease patterns with n convex faces that constrains $O(n^3)$ conditions on the layer orders between pairs of overlapping faces, and prove that it is equivalent to the established pointwise definition. We use this formulation to show that (1) such a facewise layer order can be verified in $O(\min\{n^2 p, n^2 + m p^2\}) = O(n^3)$ time, where m and p parameterize the complexity of the folding; and (2) all valid folded states of a crease pattern can be implicitly computed in $O(n^3 + \sum(ni)^3 2^{-n}(si))$ time and $O(\sum si)$ space, where ni and si parameterize a decomposition of the problem into k independent components. Lastly, we prove that unassigned crease patterns on n faces can have at most $2^{-n}(O(n^2))$ folded states, while there exist assigned crease patterns on square paper that have $2^{-n}(\Omega(n \log n))$ folded states.

Keywords: Flat-foldability; Folded states; Crease patterns

An origami Universal Turing Machine design

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It has been known since 1996 that deciding whether a collection of creases on a piece of paper can be fully folded flat without causing self-intersection or adding new creases is an NP-Hard problem (Bern and Hayes). In their proof, a binary state was implemented as a pleat, with the state corresponding to the pleat layering order; states then interact via pleat intersections. Building on some of the machinery of their result, we will present a method for constructing an origami NAND logic gate, leading to a theoretical origami Universal Turing Machine.

Keywords: Origami; Computer; Turing-complete; Universal Turing Machine

A Layering Algorithm for Flat Modular Origami

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A layering algorithm for flat modular origami models is presented. The need for drawing models automatically generated by a computer program was the driving force for this work. The three issues are discussed: (i) how to represent a modular form, (ii) how to provide information about locking of modules, (iii) how to generate the layering for a model. While the algorithm was developed independently, the core part of it is equivalent to the Flat Folder developed by J. Ku [Ku 23] for single-sheet origami models. The main difference is that instead of a graph proposed by J. Ku, the specific configurations of the folded form are translated into a set of logical clauses of 0 order and the layering is generated by a simple inference system. The Flat Folder aims to find all possible variants of layering for a single-sheet origami, while drawing a modular form was the rationale of this article. That resulted in another approach to the problem: resolving certain ambiguity of layering is a part of the design. In the same way, the position of crease lines in the CP is a part of the design process. So the algorithm takes as the input data not only CP and the folded form of a model, but also partial specification of layering, especially for flaps in pockets of joined modules. Such an approach avoids the exponential complexity of the algorithm and the layering is generated in the polynomial time. The algorithm is structured for better performance in two ways. First, the part of lower numerical complexity is applied, then if it fails, a part of higher complexity is applied, and partial results improve its performance. Second, in the typical case of identical modules, the layering problem for the complete model may be decomposed into layering of a single module next replicated for all modules and followed by layering the overlapping parts of different modules. Finally, the numerical issues generating false results for testing the geometric structure of the folded form are discussed and solved by the application of "geometry of fat lines."

Keywords: Flat origami; Layering; Rendering with hidden lines removed

Folding a Strip of Paper into Shapes with Specified Thickness

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Computational origami design typically focuses on achieving a desired shape of folding, treating multiple layers of paper like a single layer. In this paper, we study when we can achieve a desired shape with a desired constant number of layers throughout the shape, or a specified pattern of layer thicknesses. Specifically, we study the case of a rectangular strip of paper, which is the setting of the first universal computational origami design algorithm [SoCG'99]. Depending on the generality of the target surface and on the number of layers modulo 4, we give a variety of universal design algorithms, polynomial-time decision algorithms characterizing what is possible to fold, and NP-hardness results.

Keywords: Origami; Folding; Layers; Algorithms; Hardness

Kinegami: Open-source Software for Creating Kinematic Chains from Tubular Origami

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Arms, legs, and fingers of animals and robots are all examples of "kinematic chains" mechanisms with sequences of joints connected by effectively rigid links. Lightweight kinematic chains can be manufactured quickly and cheaply by folding tubes. In recent work [Chen et al. 23], we demonstrated that origami patterns for kinematic chains with arbitrary numbers of degrees of freedom can be constructed algorithmically from a minimal kinematic specification (axes that joints rotate about or translate along). The work was founded on a catalog of tubular crease patterns for revolute joints (rotation about an axis), prismatic joints (translation along an axis), and links, which compose to form the specified design. With this paper, we release an open-source python implementation of these patterns and algorithms. Users can specify kinematic chains as a sequence of degrees of freedom or by specific joint locations and orientations. Our software uses this information to construct a single crease pattern for the corresponding chain. The software also includes functions to move or delete joints in an existing chain and regenerate the connecting links, and a visualization tool so users can check that the chain can achieve their desired configurations. This paper provides a detailed guide to the code and its usage, including an explanation of our proposed representation for tubular crease patterns. We include a number of examples to illustrate the software's capabilities and its potential for robot and mechanism design.

Keywords: Origami; Tubular origami; Kinematic chains; Open-source tool

Equilateral triangle map folding

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In this paper, we extend multiple results previously known for map folds on rectangular grids, to maps on an equilateral triangle grid. Specifically, we show that it is always possible to fold a 1D equilateral triangle strip in the general folding model, while it is (strongly) NP-complete to determine whether various subsets of an equilateral triangle grid can be folded via various simple folds; in particular for assigned polyiamond paper in all models, and for assigned convex paper and unassigned polyiamond paper in the some- and all-layers models.

Keywords: Map folding; Equilateral triangle; Simple folding; NP-hard

Validity of Boundary Orders in Flat-Folding 1-Diagonal Grid Patterns

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In this paper, we investigate a particular variation of the valid order problem, which is derived from the map folding problem. Our focus is on folding a grid pattern augmented with half of the diagonal creases to form a regular grid pattern via simple folds. The conclusion is that, given an overlapping order for all the boundary triangular faces of the grid pattern, it is possible to determine in O(m + n)2 time whether a simple folding process can achieve a compatible flat-folded state, with the boundary triangular faces overlapping in the given order.

Keywords: Boundary orders; Principal diagonal grid pattern; Simple fold; Algorithm

An Algebraic Approach to Layer Ordering Constraints for Origami Flat-Foldability

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An existing algorithm [Akitaya et al. 24] computes all valid facewise layer orders of a given crease pattern by (1) decomposing the problem into smaller independent subproblems in polynomial time, and (2) solving each subproblem in exponential time. In this paper, we improve on part (1) of this algorithm by using additional polynomial-time methods to reduce the size of independent subproblems, which can yield exponential improvement in the evaluation of part (2). We do this by first recognizing that, when layer orders are taken as elements in the two-element field F2, constraints that are linear can be solved efficiently. We can also extract linear constraints from the non-linear constraints to reduce problem size further. Lastly, we can use linear transformations to maximize separation into subproblems, thus minimizing subproblem size. Finally, experimental results show the practicality of our approach, demonstrating not only a substantial reduction in subproblem size leading into part (2), but also a reduction in computation time compared to part (1) of the original algorithm for many examples.

Keywords: Flat-Foldability; Layer Order; Algebraic; Boolean Satisfiability

An Origami Simulator for Papers with Nonzero Thickness and Its Application to Support Folding Nonelementary Origami Works

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This paper presents a system to support people who want to fold nonelementary origami works. It is based on an origami simulator that can handle papers with nonzero thickness. An origami paper is initially modeled as a rigid body in a dynamics simulator. The user of the system can add a crease to the paper to split the rigid body into two rigid bodies connected by springs and dampers. Then the paper can be folded along the crease. Repeating this process, nonelementary origami works like a dragon and a humanoid robot can be folded in the system with mouse operation. After an origami work is completed in the simulator, its crease pattern is obtained. The pattern is added to an origami paper by a cutting plotter, to make folding an actual origami work much easier. The entire folding process in the simulator is recorded and can be replayed for step-by-step folding instructions as 3D graphics. Smartglasses can be used for AR display of the graphical instructions that are hands-free operable via voice commands.

Keywords: Simulation; Rigid-body dynamics; Human support

An Efficient Enumeration of Flat-Foldings: Study on Random Single Vertex Origami

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This paper deals with themes such as approximate counting/evaluation of the total number of flat-foldings for random origami diagrams, evaluation of the values averaged over various instances, obtaining forcing sets for general origami diagrams, and evaluation of average computational complexity. An approach to the above problems using a physical model and an efficient size reduction method for them is proposed. Using a statistical mechanics model and a numerical method of approximate enumeration based on it, we give the result of approximate enumeration of the total number of flat-foldings of single-vertex origami diagram with random width of angles gathering anound the central vertex, and obtain its size dependence for an asymptotic prediction towards the limit of infinite size.

In addition, an outlook with respect to the chained determination of local stacking orders of facets caused by the constraint that prohibits the penetration of them is also provided from the viewpoint of organizing the terms included in the physical model. A method to efficiently solve the problem of the determination or enumeration of flat-foldings is discussed based on the above perspectives. This is thought to be closely related to forcing sets.

Keywords: Computational origami; Flat-foldability problem; Computational complexity; Statistical mechanics; Phase transition

Efficient Enumeration of Rectangles in Origami Design

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We enumerate all convex rectangles that each of them consists of two unit angle triangles connected on edges. Such convex rectangles often appear in origami designs. We also generalize our enumeration problem with input a pair of n for π/n unit angle and m for m-gons to be enumerated: We use m – 2 triangles to build an m-gon such that the triangles can be a result of triangulation of the m-gon. In this paper, we observe the similarity between the problem and the silhouette puzzles, and construct an enumeration algorithm by modifying the algorithm for Nana-kin-san puzzle. The numbers of enumerated rectangles are 78 and 497 for n = 8 and n = 12, respectively. We show the rectangles for n = 8 in the appendix with the hope that it will be used as a catalog of shapes for origami designs.

Keywords: Efficient enumeration algorithm; Rectangles in origami design; Origami design

Shape Modeling of Developable Surfaces with a Curve Crease by Extending the Handle Curve Approach

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In this research, a system for designing curved folding interactively on a computer is proposed. This system is an extension of an existing system for designing curved folding using a curve similar to the crease, called a handle curve. Our new proposal of a sub-handle curve can be used to create a curved folding in which all the rulings do not intersect instead of creating gaps in the crease pattern. There is a tradeoff between the gaps in the crease pattern and the intersection of the rulings, and the user can easily manipulate this relationship through the distance from the crease that is set when creating the sub-handle curve. As a result, it is now possible to design curved folding that includes inflection points and uses a twisted curve as the crease.

Keywords: Shape modelling; Curved folds; Developable surface

Optimal Simple Fold-and-Cut of a Polygonal Line

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We investigate a natural variant of the fold-and-cut problem. We are given a long paper strip P and a polygonal line, which consists of a sequence of line segments, drawn on P. We cut all the line segments by one complete straight cut after overlapping all of them by a sequence of simple foldings. Our goal is to minimize the number of simple foldings to do that. When the polygonal line satisfies certain geometric conditions, we can find a shortest sequence of simple foldings for the given polygonal line that consists of n line segments in $O(\min\{n^3, n^2\log(l_{max}/l_{min})\})$ time and $O(n^2)$ space, where l_{max} and l_{min} denote the maximum and minimum lengths of the line segments.

Keywords: Fold-and-cut; One straight cut; Combinatorial optimization; Dynamic programming

Algorithmic transitions between parallel pleats

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In this paper, we present an algorithm for constructing a universal flat foldable transition between arbitrary sets of parallel pleats across a diagonal ridge crease, as well as prove the conditions required for this transition. We then demonstrate how this can be used to terminate dense bouncing in uniaxial crease patterns. Finally, we present applications of both the dense bouncing termination and the transition structure itself for representational origami design.

Keywords: Algorithms; Crease patterns; Uniaxial origami design; Dense bouncing; Level shifters

Pseudohistories of the Origin of Origami in Japan and Europe

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While many origami books include sections on the art's history, the majority of these narratives lack thorough examination of the subject. Notable investigations by scholars such as Okamura into the history of Japanese origami often receive limited attention beyond Japan. In addition, despite the efforts of researchers like Lister and others, our understanding of the European origami tradition remains inadequate. Consequently, pseudohistories regarding the origin of origami persist both outside and inside the origami community. This paper delves into the theories surrounding the emergence of origami in both Japan and Europe, and through critical evaluation demonstrates that several of these theories, particularly those asserting that origami originated as early as the fifteenth century or before, lack historical validity and should be considered pseudohistories.

Keywords: History; Pseudohistory; Origin of origami

"Origami" as a Universal Term: Akira Yoshizawa's Perspective

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¹ Japan Origami Academic Society, Japan

In 1958, Lillian Oppenheimer founded the Origami Center and proposed the now international term "origami" for paper folding. At the same time, Akira Yoshizawa of Japan was also attempting to introduce "Origami" to the world. Although these two influential people used terms with the same spelling, their intended meanings differed. Therefore, this study aims to clarify Yoshizawa's meaning for "Origami," along with its background, and to examine Yoshizawa's Origami activities in social education. The research proceeded with an analysis of literature, including Origami groups' newsletters, and the results showed that Yoshizawa consistently used Origami as a vehicle for his peace campaign.

Keywords: Akira Yoshizawa; Lillian Oppenheimer; Adult Education; History; Origami Culture

Decision making in an Origami Archive – A story of the British Origami Society Library

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In the nearly 60 years since its creation, the British Origami Society has amassed a vast collection of 'origami stuff'. With items ranging from an original copy of Senbazuru Orikata to the knitting pattern of the societies logo, the size and variety of different items in this archive present a series of significant challenges. The archives of the society are an educational resource which are preserved to protect the history of Origami, this paper presents a reflective analysis of the first phase of a decade long project to sort, condense and share this collection; providing insight into the challenges faced and decisions made to date. This paper presents several of the challenges relating to use of these resources and looks at the historic approaches taken and how these have had to change in the past few years.

With the society being first and foremost a charity, set in a backdrop of a changing financial world, ensuring the effective use of limited resources is fundamental consideration at every step. Decisions as to which items are historic resources worthy of preservation are both subjective and complex, while the cost of storage puts pressure to also provide tangible output to the wider charity. Adding to this the fragile nature of many of the items collected, a core consideration is the balance between preserving items and enabling access to them, in particular at what point the risks of handling items outweigh the risks of keeping them forever and gaining nothing from them.

The paper also looks at several other challenges including; risks this presented by engaging volunteers to support archiving tasks, the challenges of digitisation of these resources, copyright conflicts in making digital records available, and finally how to value such a collection for insurance purposes. Although we may not present answers to all of these questions, it is hoped that this reflective analysis will be useful for other organisations in similar positions. The paper concludes by looking to the future, with several other large collections held around the world, it suggests that more work is needed in joint working to share learning, and maximise the positive impact that these collections can have.

Keywords: Origami History; Archiving Origami; Copyright; Digitisation; Education

Making some geometric models

Jun Maekawa¹

¹ Origami artist, Japan Origami Academic Society, Japan

Participants make several models using materials provided by the lecturer. The models are simple but geometric and puzzle-like. These include new work, and models published in "The Art & Science of Geometric Origami".

Interlocking modular origami (wireframe) design with geometry creation and optimisation tools

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¹ Independent, Australia

Interlocking origami is a relatively niche field with few established principles for designers to use. A tool has been created which extends the 3d modelling program "Blender", enabling a designer to rapidly create and modify a complex virtual model. In addition, the tool can optimise the model for structural integrity and convert it into foldable crease patterns for each unit.

Principles and Applications of Kusudama Design

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¹ Monta Vista High School

The workshop "Principles and Applications of Kusudama Design" will explore the art and structural engineering behind kusudama, a form of modular origami that forms a structure in the shape of a platonic solid. Participants will gain an understanding of pocket and flap connections, which are the foundation of all kusudama designs. Then, participants will investigate the structural integrity and stability of kusudama models. The workshop also explores real-world applications of kusudama in engineering. In particular, one promising application of kusudama is the development of rapid-response emergency shelters, which will be discussed. The workshop uses both theoretical insights and practical activities to demonstrate the potential of kusudama origami principles in modern engineering solutions.

Parallel Session IV: Design & Education

New techniques in hex pleating for representational origami design

Brandon Wong¹

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We present new developments building off the work of Lang et al. in hex pleating as a method for designing representational origami models, including both uniaxial and non-uniaxial hex pleating. Compared to box pleating, hex pleating offers greater flexibility of layer distribution, better ability to incorporate structural textures, and a more organic appearance. We will describe how to pack basic uniaxial trees with hex pleating, how packings can be modified to redistribute paper thickness throughout the model, how to incorporate level shifters and 2D patterns, and finally how to create representational polyhedra with nonuniaxial and non-flat foldable hex pleating.

Keywords: Representational origami design; Hex pleating; Origami design methods

Symmetry Requirements and Design Equations for Origami Tessellations

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Twists on the front and back of the paper are related by more than simply turning the paper over - the transformations in pleats that lead to ever-larger twists eventually lead to twists on the other side of the paper when done in reverse. Given two fixed pleats, these transformations in each shape of grid-based twist produces an evenly spaced line of twist centers, defining the number line of that particular twist shape. Different twist shapes will have different distances between a closed twist on the front of the paper and a closed twist on the back of the paper along this number line depending on the angle between the two fixed pleats. Hybrid twists (ones that combine pleat properties of different sizes of twists of the same shape) have centers that may deviate from this number line. In certain applications of symmetry on certain tilings, the relative locations of twists (on the number line of twists) that occupy distinct tiling positions can be used to predict the twist that will occupy the final tiling position. This is useful for navigating the combinatorial explosion of potential tessellation designs while avoiding the twist(s) on the number line that require precreasing before the final collapse of the pattern. This paper will explore the number lines of square, hexagon, and triangle twists and their use in design equations on the 4^4 and $(3.6)^2$ tilings.

Keywords: Tessellation; Design; Symmetry; Twist-based; Tiling

On the Constructions of Generalized Offset Pythagorean Stretch Patterns

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We explore diverse pattern constructions based on generalized offset Pythagorean stretch (GOPS) within box-pleated designs. Our primary concept centers on the partitioning of overlapping regions of the rectilinear rivers into smaller rectangles or polygons, each corresponding to a component of the compound pattern. We also present more details on constructing the joins of gadgets, which was left out in our prior work.

Keywords: Box pleating; Pythagorean stretch; Kamiya pattern

Exploring criteria for designing novel waterbomb tessellations using triangular convex polygons

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² University of Melbourne, Australia

Waterbomb style tessellations have been explored in the past by artists such as Ronald D. Resch, Benjamin Parker and Mitya Miller. Generalised waterbomb tessellations are still underexplored in origami design. We have explored various sets of criteria for generalising waterbomb tessellations in order to enumerate valid patterns. We only consider triangular waterbomb tessellations, other polygons will be explored in future papers. In our search we have uncovered some new waterbomb tessellations, which could offer new uses in representational and geometric origami design. We conclude by discussing foldability properties and possible generalisations.

Keywords: Waterbomb-style tessellations; Corrugation; Geometric origami; Generalisation

Flat-back 3D gadgets in origami extrusions completely downward compatible with the conventional pyramidsupported 3D gadgets

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An origami extrusion is a folding of a 3D object in the middle of a flat piece of paper, using mechanisms called 3D gadgets. As examples of 3D gadgets, the conventional pyramid-supported ones generalizing the cube gadget are known, which create a top and two adjacent side faces of a frustum using two pleats. In this paper we present new flat-back 3D gadgets which improve the conventional ones in several respects. In particular, for any conventional gadget there are an infinite number of our gadgets downward compatible with it. Also, we can change the directions of the pleats in a wide range.

Keywords: Origami extrusion; 3D gadget; Frustum; Downward compatibility; Ruler and compass construction

Hybrid Hexagon Twist Interface

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The common hexagon twist options when folding grid-based origami tessellations are closed and open hexagon twists with mountain folds of pleats coming directly to the center or one grid spacing away from the center, respectively. Allowing each pleat in a hexagon twist to be positioned independently in an open or closed state provides 12 additional collapse possibilities without adding new creases to the paper beyond the central hexagon of the open hexagon twist. These hybrid options between the closed and open hexagon twists can be folded directly on a grid without marking additional reference creases and they also represent an opportunity for highly reconfigurable mechanisms due to the limited number of folds that give rise to all 14 total options. This paper will also demonstrate a method for direct substitution of hybrid hexagon twists into broader tessellation patterns and showcase the tessellation design space opened up by these new twist options.

Keywords: Tessellation; Hexagon; Twist; Reconfigurable crease pattern

Generating Smocking Patterns of Twist Folds for Clothing Design

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This paper proposes a method for generating smocking patterns of twist folds for clothing design. Smocking is a handcraft technique of sewing and making pleats over cloths, and has been used to produce decorations over the surface of cloths. However, it is not easy to generate the pattern for the desired shape on the surface of flexible cloths. For the type of smocking which corresponds to twist fold in Origami, we propose to sew down the intersection of folded pieces of cloth so that the formed shape can be kept on the cloths, and propose a method for generating smocking patterns of twist folds. The proposed method is based on the geometric transformation of polygons in crease patterns, and its geometric characteristics are clarified. The proposed method is implemented using GeoGebra, and it is validated in terms of a newly designed smocked clothing based on the generated patterns.

Keywords: Smocking; Twist Fold; Origami; Geometric Transformation; Design Support

Twist Fold Modules for Combinatorial Design of Petaloid Smocking in Clothing

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Petaloid smocking is made of cloths by sewing and flattening pleats to create shapes over the surface. We have proposed a design framework for combinatorial petaloid smocking by preserving labels of fold lines in crease patterns. However, besides rather limited size of cloths, it was difficult to design flat foldable crease patterns for large size of cloths. This paper proposes to define twist fold modules for petaloid smocking, and to design crease patterns by joining the modules. We show geometric characteristics of crease pattern based on the defined modules, and propose a design framework for combinatorial design of petaloid smocking based on the tiling of the joined modules. By designing crease patterns with GeoGebra, the proposed approach is validated via clothes making decorated with petaloid smocking.

Keywords: Petaloid Smocking; Twist Fold; Module; Crease Pattern; Tiling

Visualizing Petaloid Smocking based on Rotation of Decorations and Pleat Length

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This paper proposes an approach for drawing crease patterns based on the rotation of decorations and pleat length so that the folding process of petaloid smocking, which have been used as decorations over cloths, can be visualized. Visualization of petaloid smocking would be effective for avoiding the situation where the designed petaloid smocking cannot be actually produced due to the overlap of decorations, and to facilitating the production of petaloid smocking over flexible cloths. We show geometric properties of the crease patterns by the proposed approach, and suggest a design framework with several illustrating examples. By designing crease patterns using GeoGebra, the proposed approach is validated through the application to fashion goods. The shape of the produced fashion goods decorated with petaloid smocking can be transformed through folding and unfolding of cloths to change its size.

Keywords: Petaloid Smocking; Twist Fold; Visualization; Rotation of Decorations; Pleat Length

Comparing Twist Pattern Design Method and Design Methods of Primal-Dual Tessellations

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The twist pattern design method generates crease patterns comprising patterns known as simple flat twists. To deepen the understanding of the method, this paper reinterprets its design process following an approach that transforms plane polygon tiling by using its dual graph. Three independent methods are known to follow this approach, which generate central twist tiles, offset twist tiles, and a shrink-rotate tessellation, respectively. These generated tessellations are categorized as primal-dual tessellation. By comparing these four methods, it was revealed that the process of the twist pattern design method with an additional parameter encompasses the other three methods.

Keywords: Computational origami; Origami tessellation; Primal-dual tessellation; Tiling

Fabric Origami Tessellation: From Sewing Patterns to Crease Patterns

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This paper explores diverse algorithmic approaches for creating fabric origami tessellations, aimed at producing art pieces with unique tactile qualities and dynamic light-and-shadow effects. It primarily focuses on an innovative technique termed "tessellation grafting," used for designing sewing patterns in fabric origami. This technique originates from James Clark Maxwell's research on reciprocal figures, force diagrams, and graph duality. The paper showcases multiple instances of fabric origami crafted using "tessellation grafting." It then delves into algorithms developed to translate these sewing patterns into origami crease patterns. These crease patterns not only validate the effectiveness of the "tessellation grafting" method in fabric origami but also serve as essential guides for creating paper origami tessellations.

Keywords: Fabric Origami; Sewing Pattern; Crease pattern; Duality; James Clark Maxwell; Tessellation; Algorithm

A Systematic Notation for Pleat Systems

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Origami tessellations are often composed of the intersections of flat pleats. In this style, there are an infinite number of possible twist systems. This paper presents a notation system for uniquely describing these systems on a triangle grid. After providing the notation structure, it outlines pleat operations and how the notation describes them. The paper also identifies sets of pleat systems that have common properties, and analyzes how applying the above operations affects those properties. This delineation will provide readers with useful techniques for analyzing twists in a tiling context, or as an isolated system of folds.

Keywords: Origami Tessellations; Pleat Intersections; Origami Twist; Notation System; Pleat Operations; Triangle Grid; Abstract Algebra

Exploring participant perspectives through a collaborative art project in a university origami mathematics course

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This paper presents an exploration of a collaborative Origami art project within an open-enrollment elective Origami art and mathematics course at a regional US public university. As part of the course, students work on a culminating project that requires them to collectively plan and develop a largescale Origami art installation using skills and knowledge from the course. Details are shared on the unique structure of the collaborative art project and an exploration of the impact on participating students. Qualitative evidence gathered through written reflections revealed the development of core soft skills, including creativity and teamwork/collaboration. Findings suggest value in the collaborative Origami art project approach in a university course environment with additional studies needed to examine the impact formally.

Keywords: Collaborative project; University; Origami; Art; Mathematics; Creativity

Origami and Video Production in Mathematics Education

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This article presents research developed with mathematics teachers in initial and continuing education when working with origami and video production. The participants developed activities related to Origami Geometry, involving Basics Origami Operations (BOO's), Haga's Theorem, and aspects of video production. Through experimentation with folds in paper and the production of videos about origami, the participants produced mathematical knowledge, since it was observed that during interactions in the course, they argued using folds in paper to defend a mathematical idea and represented creases and their mathematical meanings in different ways, such as verbally, through diagrams, algebraic expressions, among others.

Keywords: Mathematical proofs; Geometric knowledge; Multimodal; Geometry of origami

Euclidean, Spherical and Origami Geometry: possibilities in Mathematics Education

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In this article we discuss how Geometry can be constituted and some of its different forms of representation. To achieve this, we present historical and conceptual aspects seeking to address how mathematical objects are treated from three different perspectives: Euclidean, Spherical and Origami Geometry. We argue that Geometry is constantly developing and, therefore, it is important to debate these issues in Mathematics Education. To this end, we will focus on possibilities to discuss the axioms through manipulable objects and technologies, aiming to contribute to the classroom and promote the breaking of a fragmented view of knowledge.

Keywords: Mathematical objects; Technologies; Paper Folding; Non-Euclidean Geometry; Geometry teaching

Enhancing the teaching of visuospatial skills in elementary education through professional learning and implementation of a unique origami curriculum

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This study addresses the experience of teachers in the under-researched area of visuospatial development for elementary skill education, utilising case studies that consist of semi-structured interviews and focus groups that were recorded and transcribed to examine the experiences of elementary teachers who participated in the professional learning program, Project VisMO, an innovative origami-based curriculum. This research contributes to understanding the impact of professional development on teaching visuospatial skills and underscores the potential of origami-based learning as a tool for improving educational practices, highlighting areas for future research in sustainable instructional strategies.

Keywords: Visuospatial skills; Teacher professional development; Origami-based curriculum; Elementary mathematics

Educational Origami: The Design and Implementation of the Course 'Mathematical Paper Folding' in Upper Secondary School

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The Estonian upper secondary school curriculum emphasizes mathematics as investigative and research-based, fostering diverse teaching methods for analytical thinking. This paper details the design process of the elective course "Mathematical Paper Folding," aligning with national curriculum competences. Integrating origami's ancient knowledge, the course explores fundamental folding techniques, linking mathematical relationships to practical applications. The design not only achieves mathematical learning outcomes but also enhances motor skills, spatial competence, and design thinking. Student feedback guides ongoing improvements, highlighting the course's impact on multifaceted skill development.

Keywords: Educational origami; Upper secondary school; Mathematical outcomes

Exploring Origami's Impact: Nurturing Spatial Skills to Enhance STEM Access among Individuals with Blindness

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The potential for Origami to enhance the development of spatial knowledge among children is generally recognized. However, limited knowledge exists regarding its effectiveness among individuals with blindness, as well as its inclusive or exclusive aspects within this unique demographic. This exploratory case study focuses on two individuals with blindness who demonstrate exceptional spatial knowledge and a strong affinity for origami. The study aims to deepen the understanding of origami's potential in enhancing spatial skills in individuals with blindness, while also exploring various physical aspects of origami that could increase accessibility for this population.

Keywords: Origami; Spatial skills; Individuals with blindness

Approaching proof in geometry by folding problems with pre-service teachers

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This study focuses on the geometric thinking of first-year pre-service middle school mathematics teachers and the potential contribution of folding problems to improving their geometric thinking regarding proofs. A significant proportion of these students do not work with the level of geometric thinking that might be expected: When asked to prove a geometric property, they base their argumentation partly on observations or measurements on the geometric diagram, whereas an answer based solely on deductive reasoning is required. To foster the entry of these students into deductive geometry, while at the same time offering to every student a first discovery of concepts related to the teaching and learning of plane geometry, we designed and experimented a teaching sequence based on folding problems.

Keywords: Proof; Geometry; Trigonometry; Middle school teacher education; Folding problems

Simplifying the Fold-and-One-Cut Problem: A Pedagogical Approach for Origami Engineering Education

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The fold-and-one-cut problem, often linked to Houdini's Magic trick, involves making paper cutouts with one straight cut. While the literature leads us to valid crease patterns, it lacks a straightforward teaching method, in particular for the assignment of mountain and valley folds. This paper presents a simplified, pedagogical approach for crafting single- and multi-polygon cutouts, illustrated by examples. Emphasizing origami principles for flat-foldability, our approach transforms the fold-and-one-cut problem into an effective educational tool. Classroom implementation demonstrates the approach's efficacy in reinforcing student's understanding of origami design principles. Moreover, our approach promotes hands-on applications of the method, which can be readily implemented beyond the classroom, e.g., workshops and outreach activities.

Keywords: Fold-and-one-cut; Origami engineering education; Multi-polygon cutouts

Engaging secondary school students in building formulas based on mathematical folding

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We study the potential of mathematical folding as an invitation to mathematize what is literally in your hands. In the reported study, 16-18-year-old preuniversity students (n=19) develop formulas to describe a pattern in a folded strip. As part of design-based research, we designed a lesson based on the theories of didactical situations and embodied cognition. In this lesson, there is no dichotomy between folding and mathematizing; teacher-guided progressive mathematization and a gradual introduction of mathematical notation are fostered, as well as language derived from the actions and folded artifacts of the students. Students reached multiple but different mathematical layers of the folding tasks in the studied implementation. We conclude that this makes the lesson design suitable for students with different work attitudes and grades.

Keywords: Mathematical folding; Secondary education; Embodied cognition; Educational design research; Realistic mathematics education; Geometry; Formulas; Mathematical reasoning

Critical thinking class using Origami

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¹ Hokkaido University, Japan

Critical thinking is important to skill to expand various possibilities. Here, we will introduce how we can practice critical thinking skills using brain storming techniques and topics of Origami. Origami is not a source of innovation.

Keywords: Critical thinking; Brain storming; Origami

The use of cutting and folding techniques in the creative process by Arts and Product Design students

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For a long time, it was believed that origami and kirigami were consisted by repetitive pre-conceived steps, in a certain order, to obtain the same models. However, Akira Yoshizawa revolutionized the history of origami by creating thousands new models, as well as Josef Albers during Bauhaus (between 1920 and 1930), also realized the potential of folds and cuts as an exploratory technique and encouraged his students to manipulate paper to study transformation and development of structures in order to stimulate the development of creative thinking.

During Brazilian neoconcrete artistic movement (1950-), some artists explored cuts and folds in order to transform two-dimensional materials into a three-dimensional structure as a new search for solutions in Art. Augusto de Campos and Julio Plaza explored techniques similar to 3D kirigami in a sequence of concrete poetry paper cards named "Poemóbiles"; others applied cutting and folding techniques in rigid materials, thin or thick, articulated or not, such as the flexible sculptures in aluminum sheets called "Bichos" and "Relógio de Sol" by Lygia Clark and the sculptures by Amilcar de Castro and Franz Weissman. With these and many more recent examples, it is clear that exploring folding and cutting techniques, both on paper ("kami") and other materials, allows new interpretations, possibilities and, consequently, new challenges. However, it seems that this history was left behind, because Brazilian Design and Visual Arts students do not use folds and cuts potential to its fullest, limiting themselves by still considering origami only in its traditional sense and application.

In order to change this scenario, at least a little, this author began to teach folding and cutting techniques to undergraduate students of Arts and Product Design in a Brazilian University. During this course, we study from Masahiro Chatani and Paul Jackson's books, geometry concepts, designing process and different material studies. The results range from creation of new pop-up cards to origami sculptures, from foldable furniture to medical products. This article will present this process and some results from Arts and Product Design students during the last years and we hope it can be relevant as it presents a path that can be followed and improved by others.

Keywords: Design process; Education; Art; Product Design; Creative process

Mathigami, 6OSME Documentary, and Learning Visual Spatial Teaching in 2023

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VIDEO OVERVIEW: This particular story begins with Project Mathigami in 2013, motivated by several presentations at 50SME that highlighted the vast evolution and the extensive applications of origami at that time. Seeing the successful applications of educational programs around the world, math professor Dr. Perla Myers and educator Celina Gomez were driven to establish a movement at the University of San Diego: Mathigami. During the pilot year of the program, Project Mathigami successfully gathered a team that developed a curriculum for teaching mathematics through the art of Origami and deployed twenty-one instructors to engage over one hundred 3rd to 5th grade students. The lessons were adapted for a wide range of ages.

Mathigami allowed educators, learners, and students alike to explore origami whilst upholding an accessible, hands-on emphasis in mathematics, engineering and spatial relations, amongst other things. The Mathigami team was diverse in both age, academic degree, and area of expertise.

In 2014, a team of seven college students from Project Mathigami journeyed to Tokyo to give a presentation during 60SME They traveled with professional filming equipment courtesy of the University of San Diego to collect information and video interviews that would become a documentary.

The documentary contains interviews with eighteen of 6OSME's presenters, Miri Golan, Robert Lang, Kōryō Miura, Jason Ku, Tom Hull, Erik Demaine, Martin Demaine, Jeannine Mosely, Beth Johnson, Ali Bahamani, Goran Konjevod, Micheal Winckeler, Jun Mitani, Surynash Chandra, Chris Palmer, Gregory Epps, Jose Pardon, and Ilan Garibi. It highlights the interdisciplinary collaborations within the different fields that take advantage of origami, and emphasizes the power of origami for teaching.

WORKSHOP OVERVIEW: This workshop will introduce and feature the draft of the 60SME documentary, edited by attendee and USD participant Anna Walsh to discuss and reflect upon how what started as Mathigami has evolved over the years. The documentary will be followed by a demonstration, conversation and discussion about the VisMO curriculum, the result of an NSF grant (1920821) and a continuation of Mathigami. VisMO teaches visuospatial skills and vocabulary to elementary school kids through a more formal origami-based curriculum. Participants will be taken through an example of one of the VisMO origami lessons and shown the 3-i process (identify-imagine-investigate), a process developed by the team to engage students in developing visuospatial skills through paper play.

Keywords: Mathematics in origami and folding; Origami-based curriculum; Visuospatial learning; Origami in education

Origami and Mathematics in the classroom: Increasing Spatial Vocabulary and Decreasing Mathematics Anxiety through Paper Folding

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Visuospatial skills provide an important tool for mathematical problem solving. Visuospatial skills, defined as the capacity to perceive, retain, retrieve, and mentally transform the static and dynamic visual information of objects and their relationships (Wai, Lubinski, & Benbow, 2009; Verdine et al., 2014; Uttal et al., 2013), are an important predictor of long-term achievement and attainment in STEM (e.g., Wai, Lubinski, & Benbow, 2009; Kell et al., 2013; Uttal et al., 2013), yet little has been done to focus on the use of visuospatial skills in our educational practices, as visuospatial abilities remain a "major blind spot" in STEM education (NRC, 2006; Khine, 2017; Gagnier & Fisher, 2017). To better understand the relationship between visuospatial skills and elementary school students' understanding of mathematics, an origami-based, hands-on curriculum (Project VisMO; Visuospatial-Mathematics-Origami) was developed to study the effects of origami lessons on English Language Learners' (ELL) and non-ELL students' visuospatial skills, spatial vocabulary, and mathematical anxiety.

A total of 179 students aged 9-11 (Mage = 10.35) from Southern California school districts participated in the study. Students received 10 sessions of the curriculum, 1.5 hours each, and were randomly assigned to two conditions; the Owl group, or treatment group (88 students), received VisMO lessons, while the Elephant group, or control group (82 students), received literacy-based social-emotional lessons. Due to the onset of the COVID-19 pandemic, all lessons were shifted to online delivery. Students completed measures of mathematics, spatial skills, spatial vocabulary, mathematics attitudes and working memory before and after engaging in the curriculum, and a last time at a one-year follow-up.

The VisMO lessons showed benefits overall on elementary school students' spatial vocabulary and in reducing mathematics anxiety. Data was analyzed using a series of hierarchical linear models (HLM); Separate HLM models were run for immediate (posttest) and sustained (follow-up) impacts with each model including treatment group, gender, language, and pretest score as fixed level 1 predictor variables. Best fitting models demonstrated a significant effect of group on posttest spatial vocabulary scores (B = -0.26, t(93) = -2.10, p = 0.03), with posttest scores higher for those participating in VisMO lessons compared to the control lessons. VisMO lessons did not have an

immediate or sustained impact on math interest, but they did show an immediate impact on math self-efficacy (B = .28, t(95) = 2.39, p = .02) and math anxiety (B = .26, t(95) = 2.09, p = .04). For math anxiety, students who participated in the VisMO lessons showed lower math anxiety.

Although immediate impacts were found for spatial vocabulary and mathematics anxiety, these benefits were not sustained over the one-year follow-up period in our current analyses. However, the one-year follow up sample experienced high attrition (retaining only 30% of the original sample), resulting in a sample size that likely underpowered finding any potential sustained impacts. VisMO lessons were designed to be delivered in an in-person format, and we hypothesized that online delivery may have limited the lessons' impact. To further explore, we conducted two preliminary pilot studies that found consistent increases in spatial vocabulary and decreased math anxiety as well as newly discovered increases in math and spatial skills. Taken together, our initial results, along with preliminary pilot findings, suggest that VisMO lessons have an immediate impact on some mathematics skills and spatial vocabulary, and appear to have a larger impact when those lessons are delivered in-person. Future research with larger sample sizes at a one-year follow up are still needed to determine which impacts may be sustained over time.

Keywords: Visuospatial skills; Math anxiety; Origami-based curriculum; English Language Learners; Online learning

Vision impaired origami teaching and folding - a Melbourne experience

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Origami is not only a visual art, it is also very tactile, and it has long been recognized that blind people enjoy origami. In recent years, the Melbourne Origami Group not-forprofit has been collaborating with the Blind Sports and Recreation Victoria charity to help their members fold origami for annual origami Christmas tree events. Our collaboration has yielded useful observations not only into methods of teaching the vision impaired, but also on the social and mental well-being impact for those who participate. Our experiences may yield useful insights to help others pursue similar collaborations.

Folding all 4x4 Rotationally-Symmetric Diagonal-Grid 2-Color Patterns

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From 2022-23, Brown designed foldings and folding sequences for all 354 rotationally symmetric 4×4 diagonal-grid 2-color patterns. Ku drew crease patterns for the designs and verified their validity. This paper discusses the various approaches taken to begin each new solve, techniques for working toward a solution via active folding (rather than planning an entire solve beforehand), and evaluation of the final result, using both objective measures such as efficiency and seamlessness, as well as subjective measures such as ease of the folding sequence, layering, and locking.

Keywords: Color change; Rotational symmetry; Seamless

From A4 Paper to Tangram Puzzles: the Math Behind Paper Folding

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Tangram is both a popular Chinese puzzle and a useful teaching tool for K-12 math teachers. Due to the inconvenience of carrying around and easy missing some of the parts, people are more tend to make tangram by cutting or by origami a sheet of paper. In this paper, we introduce two established methods for folding a tangram using either A4 paper or square paper. Subsequently, we extend these techniques to accommodate standard paper sizes such as Letter or Legal. This improvement is particularly advantageous for individuals in regions where A4 is not the standard size. The accuracy of the novel tangram folding methods can be mathematically demonstrated.

Keywords: Tamgram origami; Mathematics teaching; Silver ratio

Folding Fashion: Origami for Minimal Waste Garment Construction

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It is estimated that 92 million tons of textile waste is produced annually. From that waste, 15-20% is the result of the excess fabric remnants that are left behind after a pattern design has been cut from a sheet of fabric. By applying an origami crease pattern to a square of fabric, and folding and stitching the fabric into the designated fold pattern, we can create a textile item that circumvents the traditional method of pattern cutting and the fabric waste that is created as a byproduct. We demonstrate the fold-patterning technique in the development of handbags, and plan to extend the techniques application to the creation of garments. This process involves abstracting geometrical constructions together with origami design principles (such as developability and vertex flat-foldability), which are not considered in isolation, but in connection with their actual implementation into fabrics and textiles. Through this research, we aim to contribute to the ongoing dialogue on sustainability by emphasizing the potential of this origami-inspired approach to reduce the waste byproduct of fabric scraps while introducing a new aesthetic dimension to garment design.

Keywords: Sustainability; Textile design; Fold-patterning

Rotational origami of polyhedral type

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There are various methods for appoximating a surface of revolution by origami. Among them, the rotational origami method using curved folds has been made progress by Palmer, Lang, Mitani, and others. The resulting origami model consists of approximating cylindrical surface segments (called gores) circumscribing the surface of revolution, each of which involves a flange outside. From the viewpoint of approximation, we want the area of the flanges as small as possible. In this paper, we apply the rotational method to not a flat or cylindrical but polyhedral piece of paper, to reduce the area of the flanges.

Keywords: Rotational origami; Polyhedral type; Surface of revolution; Curved folding; Reduction of flanges

Multifaceted Dialogue in an Origami-Based Art Process

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We present two approaches to creating origami-based artworks through the examples of two works. The creative process of the first work begins with a traditional square sheet of paper, while the second approach combines traditional origami techniques with a methodology that originates from the idea. Symbolic meanings of both approaches will be discussed.

Keywords: Experimental folding; Miura fold; Waterbomb origami; Artbased research

Pillow Box Design

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This paper focuses on packaging design using origami techniques, specifically designs incorporating curves, known as pillow boxes. While conventional paper packaging boxes are typically cuboid, pillow box designs include curved surfaces, offering both aesthetic and practical advantages. This study analyzes the specific curved folds of pillow boxes, clarifying the fundamental geometric condition these curves must meet. Additionally, it proposes new design variations for pillow boxes based on the condition. The relationship between the shape of the folds and the volume of the final threedimensional shape is also explored. This research extends the boundaries of functionality and aesthetics in origami design and explores new possibilities in packaging solutions.

Keywords: Curved fold; Pillow box; Volume

Folding curves over pleats

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A curved crease on a sheet of paper causes the paper on both its sides to curve. I show how to fold a class of models that seem to defy this with most of the paper flat while a curved ridge protrudes from the surface. By "show how to fold" I mean explain the physical manipulation of paper. What actually happens during the folding is unclear, as are the details of the exact geometry of the result. While for me the primary motivation for exploring this class of models is the freedom it allows in the design process, the question of how these models actually fold is intriguing, and answering it may eventually provide further insights into the behavior of paper as a sculptural medium.

Keywords: Curved creases; Origami design; Folding over pleats; Non-geometric design methods

Topological Transformation of the Miura Ori Crease Pattern

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This paper delves into the world of Miura Ori tessellations, demonstrating how the base model can be transformed and extended into very different looking origami designs by applying various topological transformation to the original crease pattern. Central to our exploration is a framework for executing these transformations. While the designs can be drawn digitally using graphic design software without the use of mathematics, we have developed a mathematical method based on the use of complex numbers and polar radial functions. This method enables the transformations to be performed with precision while delivering the output with efficiency. The techniques outlined here are then generalised to other origami corrugation crease patterns such as the (curved) Triangle Fold pattern by Ron Resch, and coded for ease of application, offering a realm of creative possibilities for designers.

Keywords: Miura Ori; Origami corrugation design; Origami software

Exploring Collapsible Origami Structures for Beehives, Portable Toilets and Wheelchairs - Challenges and Opportunities in Design Innovation

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While origami applications continue to revolutionise products around the world, crosscutting a plethora of sectors and industries, there are still many everyday items for which folding variants have not been introduced. This poses the question of which applications may be appropriate for folding and why. In this paper we explore the application of simple rigid folding techniques to several seemingly obvious items. Focusing initially on challenges arising from creating a collapsible beehive, we explore factors impacting design choices for collapsible boxes which were crucial in creation of a product which functions at least as well as solid alternatives.

A beehive presents itself as relatively simple structure formed of stacking frames, and there are a number of ways in which its component pieces can be folded flat. The benefits of a foldable hive range from storage and transportability to sustainability and safety, however the added complexity of a hinge or a crease, in an area where the simplicity of the traditional solution is its advantage, presents a barrier to design innovation. Factors such as strength, durability, warmth and materials are also key in the successfully functionality of the end product. This paper presents several key factors which were important in the design of an origami beehive, and discusses how these factors impacted the final design of the our prototype hive.

The paper then explores how these factors translate into other products, in particular looking at whether similarities can be seen when the same approach is applied to other products. The portable toilet presents a very similar challenge to the beehive, and potential impact of a viable solution could be huge. The paper looks at the differences and overlap between the relevant factors for each design, suggesting for example that warmth requirements of hives may mirror the privacy requirements of portable toilets as they both relate to the accuracy/tolerances in hinged elements. Although seemingly new factors such as hygiene arise, these are shown to be relatable to the original beehive problem. As a final and significantly different example the paper explores if these factors may also relate to the design of foldable wheelchairs, looking at recent innovations in folding chair design and the advantages and weaknesses of folding chairs.

Keywords: Beehive; Collapsible Beehive; Origami Beehive; Folding Portable Toilet Design; Origami Wheelchair

Folding investigations led to the design of multiplanar tessellations linked by circular 4 colouring permutations. Folding across this array of geometric coloured glyphs results in 3D enantiomorphic, non-numeric "dice"

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Experiments to enclose volume by folding any polygon have prompted the creation of square-based 2D tessellation designs with links to Nubian tilings and a floor-tiling pattern designed and used by James Clerk Maxwell. Locating a cube folding pattern on either of two different kinds of vertices within a monochrome tessellation gives rise to different all-over patterned cubical forms.

There are 6 ways to colour the basic monochrome 2D tessellation in four colours by exploiting all three circular colour permutations and their mirror images. Focusing on folding across the 3D intersections of these coloured planes results in 8 pairs of enantiomorphic, non-numerical, dice-like objects. On the faces of these "dice", the concept "adds to 7" is replaced by the entanglement "is the mirror image of".

For every 4 coloured Escher or Escher-like 2D artwork, or, for example, for any four coloured digital render of (z^{4-1}) by Newton's method fractal program, by permuting the colours of areas or the basins of attraction there exist 2 more potential coloured renderings of such designs, plus mirror images of each permutation.

Templates for dice-like modules and related compound uber-dice templates can be extracted by folding from the basic 2D monochrome tessellation where they exist in spatial relationship to each other in a similar manner to how a drop shadow relates to its typeface. In 3 dimensions this drop shadow concept converts to "insideness and outsideness" illustrated further by the existence of multiple foldable crystal forms related to each other by identical volume.

Keywords: Origami; Folding; Tessellation; Non-numeric dice; Geometric dice; James Clerk Maxwell; Permutations; Four colours; Volume equivalent crystals

Posters

Generating Strings for Forming Petaloid Smocking based on Crease Patterns

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Petaloid smocking is a handcraft technique of producing the decorations of flower shapes over the surface of cloths. However, since cloths are too flexible to sustain the decorations, it is not easy to produce desired decorations over cloths by fattening and arranging pleats. Toward supporting the forming of petaloid smocking, this paper proposes to generate strings based on crease patterns of petaloid smocking. Based on the folding map of crease patterns, algorithms to generate strings to support the forming of petaloid smocking are proposed. The proposed method is implemented using GeoGebra, and is verified through the application to fashion goods decorated with petaloid smocking.

Keywords: Smocking; Twist Fold; Crease Pattern; Strings; Folding Map

Triangle-supported negative 3D gadgets in origami extrusions with a canonical correspondence to flat-back positive 3D gadgets

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Positive 3D gadgets extrude a top and two adjacent side faces of a frustum from a flat piece of paper using two pleats. Then a natural problem comes up whether it is possible to construct from any positive 3D gadget its 'negative' with the opposite unevenness by modifying the crease pattern without changing the pleats. In this paper we give an affirmative solution to this problem. For this purpose, we give a canonical construction of negative 3D gadgets with a supporting triangle on the back side, which have a natural correspondence to special flat-back positive ones among infinitely many compatible ones.

Keywords: Origami extrusion; 3D gadget; Canonical 3D gadget; Frustum; Ruler and compass construction

Truncated 3D gadgets in origami extrusions

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For a top and two adjacent side faces of a frustum, we consider its truncation by a plane passing through the top face and the bottom endpoint of the ridge, so that the cross section is a triangle. In this paper we present a construction of 3D gadgets in origami extrusions which create such a truncated 3D object using two pleats. For this purpose, we consider the range of the assembly point of the bottom vertices of the side faces, which point determines the directions of the two pleats. We also find a condition for the negative of the gadget to exist.

Keywords: Origami extrusion; 3D gadget; Truncated 3D gadget; Frustum; Ruler and compass construction

A variational approach to the paper bag problem for flanged origami packages folded from dihedrons of convex polygons

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The paper bag problem is naturally generalized to ask the maximum possible inflated volume of the 'dihedron' of any given connected planar domain obtained by gluing together its two copies along the boundaries. In this paper, given a general convex polygon satisfying a certain condition, we solve the variational problem for maximizing the volume of the flanged origami package folded from its dihedron with curved creases by the rotational origami method. We can apply our result to rectangular dihedrons to obtain origami packages with larger volume than expected by Robin's formula. As a by-product, we also obtain large-volume pillow boxes.

Keywords: Paper bag problem; Robin's formula; Variational problem; Flanged origami package; Pillow box

Designing of a Novel Umbrella Based on the Bricard Linkage

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Using the geometry of Origami and the Bricard linkage, we construct a novel umbrella design including crease patterns and the designing method. Among the mechanisms and origami patterns anticipated for application in umbrellas, we focus on the 6R Bricard Linkage, a mechanism considered by NASA for the space structures. By using 6R Bricard Linkage deploying in a flat regular hexagonal shape we create umbrella frames that deploys in three dimensions by adjusting the angle of the hinge. Our design included the implementation of a folding/unfolding and a shape keeping mechanisms. The presentation will describe the designing method the umbrella constructed by using algorithmic design tools, and present a prototype using aluminum pipes and a 3D printer. Additionally, a novel approach to ensuring a smooth folding/unfolding process for the umbrella will be discussed.

Keywords: Bricard Linkage; Umbrella; Deployable structure in three dimensions; Grasshopper

Amplifying the kinematics of origami mechanisms with spring joints

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Due to its rigid foldability and predictable kinematics, the reverse fold is the fundamental mechanism behind some of the most well known origami kinematic structures, including the Miura Ori, Yoshimura, and waterbomb patterns. However, the reverse fold only has one parameter to control its behavior: the starting fold angle. In this paper I introduce an alternative to the traditional reverse fold-based on the springinto-action pattern-called the spring joint. This novel rigidly foldable mechanism is able to couple multiple reverse folds into a compact space to amplify the kinematic output of a traditional reverse fold by up to ten times, and to add one parameter for each reverse fold, giving more programmatic control of origami structures. Methods of parameterizing both the starting angle, the path of travel, and the axis of motion are also introduced such that the spring joint can be engineered to any application within compliant mechanisms, deployable structures and robotics. Unfortunately, this versatility comes at the cost of a large buildup of layers, making the spring joint impractical for thick origami mechanisms. To solve this problem, I also introduce a modular alternative to the spring joint that has no additional layers, with the same kinematic properties. Both of these mechanisms are tested as replacements for the reverse fold in both traditional and custom origami structures.

Keywords: Origami; Mechanism; Deployable Structure; Rigidly foldable

Re-programmable Matter by Folding: Magnetically Controlled Origami that Self-Folds, Self-Unfolds, and Self-Reconfigures On-Demand

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We present a reprogrammable matter system that changes shape in a controllable manner in real-time and on-demand. The system uses origami-inspired fabrication for self-assembly and repeated self-reconfiguration. By writing a magnetic program onto a thin laminate and applying an external magnetic field, we control the sheet to self-fold. The magnetic program can be written at millimeter resolution over hundreds of programming cycles and folding steps. We demonstrate how the same sheet can fold and unfold into multiple shapes using a fully automated program-and-fold process. Finally, we demonstrate how electronic components can be incorporated to produce functional structures such as a foldable display. The system has advantages over existing programmable matter systems in its versatility and ability to support potentially any folding sequence.

Keywords: Origami; Self-assembly; Magnetic actuation; Reconfigurable robotics